

# Tombstone Transformation Functions for Ensuring Consistency in Collaborative Editing Systems

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**Abstract**—In collaborative editing, consistency maintenance of the copies of shared data is a critical issue. In the last decade, Operational Transformation (OT) approach revealed as a suitable mechanism for maintaining consistency. Unfortunately, none of the published propositions relying on this approach are able to satisfy the mandatory correctness properties  $TP_1$  and  $TP_2$  defined in the Ressel’s framework. This paper addresses this correctness issue by proposing a new way to model shared state by retaining tombstones when elements are removed. An instantiation of the proposed model for a linear data structure and the related transformation functions are provided.

## I. INTRODUCTION

Collaborative editing systems allow users to edit the same document from multiple sites across Internet. Users could be distributed in space working at different places. Depending on the work context, they can work synchronously or asynchronously. Synchronous collaboration is also called real-time editing since when a user performs some modifications on the document, these modifications are instantly sent to other users who can see them without any delay. In the contrary, in asynchronous collaboration, users may not work at the same time. They work in isolation: they can decide when to publish their modifications and when to integrate modifications performed by other users.

In these systems, the shared documents are commonly replicated at multiple sites. Parallel modifications on these copies may happen and therefore there are exposed to potential inconsistencies. One of the main issues in collaborative editing is how to maintain consistency of shared documents copies. Consistency maintenance mechanisms are classified in two categories whether they are pessimistic or optimistic.

Pessimistic approaches try to give the impression there is only one highly available copy in the whole system. Only one copy – or part of a copy – can be edited at the same time while all the copies can be read. This principle is generally realised using a locking mechanisms such as in database transaction systems [2] or in turn-taking protocols [8]. Over the years, this class of mechanisms revealed unsuitable for collaborative editing even though they ensure strong consistency. They place too many restrictions on collaborative interactions – no concurrent updates are allowed – and they do not support work disconnected from the network. Furthermore, the lapse of time during which the system asks for a lock makes these mechanisms not appropriate for real-time editing.

On the contrary, optimistic approaches [24] are more suitable for collaborative editing since they tolerate divergence between copies as much as they will converge to the same value in the end. In particular, one approach so-called Operational Transformation [6] (OT) has been specifically designed to fulfil the requirements of collaborative editing. In this approach, updates performed by one user are applied on the local copy without any delay. Next, they are broadcast to other copies whether synchronously or asynchronously. Finally, updates have to be executed on the remote copies. Incoming updates are transformed according to concurrent updates that might have been performed in the meantime on these remote copies. These transformations are computed in a way that will ensure convergence of the copies. It is worth to point out the local response time is not sensitive to network latencies since local updates are executed immediately.

Since the initial work of Ellis et al. [6], several OT frameworks [23], [28], [13], [15] have been proposed. The first OT framework was developed in 1996 by Ressel et al. [23]. This framework makes a strong separation between a generic integration algorithm and specific transformation functions. Transformation functions depend on the type of shared data, whereas the integration algorithm does not. If a developer of a collaborative editor wants to provide sharing of one specific data type, he has to write the adequate transformation functions and prove they conform to the two correctness properties  $TP_1$  and  $TP_2$ . Under these conditions, the integration algorithm will ensure that causality between operations is preserved and convergence of copies is achieved.

Unfortunately, satisfying  $TP_2$  is very difficult. It has been proven [11], [13] that all proposed transformation functions are not correct regarding this property.

A proposed solution to this problem was just to require only  $TP_1$  property and fix a total ordering on the integration of operations. The SOCT4 algorithm [29] implements this strategy while conserving the generality of the OT approach. The SO6 synchronizer [1] based on SOCT4 demonstrated the use of this OT’s strength to build a tool very similar to CVS capable to reconcile a file system containing text files and XML documents. Unfortunately, building the total ordering requires a central site or a stable pool of sites [5]. These new constraints prevent SOCT4 to be used in a pure decentralized environment such as a peer-to-peer network.

In order to escape from  $TP_2$  property, other frameworks have been proposed [16], [9], [20]. Although these frameworks are very interesting, their integration algorithms are closely related to linear-structure properties of shared document for which they have been designed. Thus, in order to handle new types of data or to allow more operations to be performed on the data, the integration algorithm has to be modified and some new correctness properties must be determined and proven on it. Hence, if they propose new ways to model and solve consistency problems, they do not have yet the generality of the original Ressel’s framework.

This paper presents the first set of transformation functions that ensures  $TP_1$  and  $TP_2$ . This new set is the transposition into the Ressel’s framework of the idea of tombstones developed in [20]. This new set of transformation functions, called TTF (Tombstones Transformation Functions), demonstrates that it is possible to instantiate the Ressel’s framework.

This paper is structured as follows. We start by presenting the Ressel’s framework. We then describe our approach and discuss its correctness. After, we relate our approach to previous works. Finally, we conclude and point some future work.

## II. BACKGROUND AND OPEN PROBLEMS

A collaborative editing system consists of a set of participant systems connected by a communication network. In the following, a participant system is called a site. There is one site per user. Usually a site correspond to one user’s workstation, however sometimes one computer might run multiple sites. Because the response time demands of collaborative editing systems are so high, and in order to support collaborative work in isolation, the shared data state are replicated at every site. In other words, a collaborative editing system is modelled as follows. It considers  $n$  sites, each site owns a copy of shared data. When a site performs an update, it generates a corresponding operation. Every operation is processed in four steps: (i) executed on one site, (ii) broadcast to other sites, (iii) received by other sites, (iv) executed on other sites.

The OT approach distinguishes two main components:

- an *integration algorithm*. This algorithm is in charge of reception, diffusion and execution of operations. When necessary, it calls transformation functions. This algorithm does not depend on type of replicated data ;
- a set of *transformation functions*. These functions merge concurrent modifications in transforming two concurrent operations in order to execute them in a serial order. These functions are specific to a particular type of replicated data such as string of characters, XML document or file system.

The study of this paper is restricted to shared document relying on a linear structure. Problems and proposed solutions can be generalised to more complex structures such as hierarchical structures as shown in [4], [18], [10]. Without loss of generality, in the remaining of this paper, the shared document is considered to be a string of characters. A string of characters might be updated by performing two kinds of operations:

- $ins(p, c)$  inserts a new character  $c$  at position  $p$  in the string.
- $del(p)$  removes the character located at position  $p$  in the string.

The first character of a string is assumed to be located at position 1.

Over the years, consistency maintenance in OT has been refined to the guarantee of two criteria: *causality preservation* and *convergence of copies*.

Considering two operations  $op_1$  and  $op_2$ , operation  $op_1$  is said to *precede*  $op_2$  if and only if  $op_2$  is generated on a copy after  $op_1$  was executed on this copy. Subsequently,  $op_2$  may depend on effects of execution of  $op_1$ . *Causality preservation* criterion ensures that all operations ordered by a precedence relation, in the sense of the Lamport’s *happens-before* relation [12], will be executed in the same order on every copy. Two operations  $op_1$  and  $op_2$  that are not related by a precedence relation (neither  $op_1$  precedes  $op_2$ , nor  $op_2$  precedes  $op_1$ ) are said concurrent.

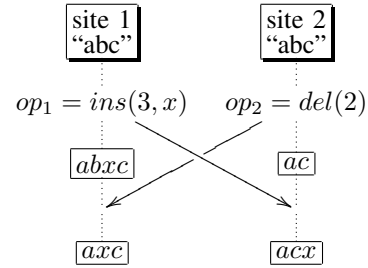


Fig. 1. Divergence problem.

Two concurrent operations can be executed in different order on two different copies. Consequently, when an operation is received on one site, the current state of shared object may be different from the one where the operation has been generated. Thus, executing this operation in its generated form on a remote site may not preserve its effects and the copies may not converge. Figure 1 illustrates such a scenario. Operations  $op_1$  and  $op_2$  have been generated concurrently on two different copies of the string “abc”.  $op_1$  inserts an  $x$  at position 3 to obtain the string “abxc”, while  $op_2$  removes the characters  $b$  located at position 2. If these operations are executed as-is when they are received by other sites, it leads to the two divergent states “axc” and “acx” as depicted in Figure 1.

In order to solve this kind of consistency problems, Ellis et al. [6] introduced a transformation function  $T$ . This function is used to transform remote operations when they arrive on a site. Remote operations are transformed regarding concurrent operations that were already executed on local copy.

For instance, in our previous example,  $op_1$  is not any more executed as soon as it arrives on site 2, but it is transformed regarding concurrent operations, in our case operation  $op_2$ . As  $op_2$  removed a character located before the insertion position of  $op_1$ , the insertion position of  $op_1$  is decreased of one position to take into account previous execution of  $op_2$ . Consequently, on site 2, operation  $op_1' = ins(2, x)$  has to be

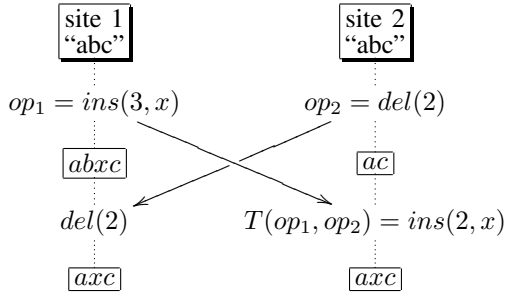


Fig. 2. Convergence by transforming.

executed (see Figure 2). Intuitively, the transformation used is defined as follows:

$$T(\text{ins}(p_1, c_1), \text{del}(p_2)) :- \\ \text{if } (p_1 \leq p_2) \text{ return } \text{ins}(p_1, c_1) \\ \text{else return } \text{ins}(p_1 - 1, c_1)$$

**Definition 1** A transformation function<sup>1</sup>  $T$  takes two concurrent operations as parameters. These two operations, namely  $op_1$  and  $op_2$ , must be defined on a same state  $S$ . The function computes a new operation  $T(op_1, op_2)$  equivalent to  $op_1$  – i.e. has the same effects – but defined on the state  $S \odot op_2$ .  $S \odot op_2$  is the state resulting from the execution of  $op_2$  on state  $S$ .

Later, Ressel et al. [23] identified two properties  $TP_1$  and  $TP_2$  which must be satisfied by transformation functions for ensuring convergence of the copies independently of the integration order of concurrent operations.

**Definition 2** For every two concurrent operations  $op_1$  and  $op_2$  defined on the same state, the transformation function  $T$  satisfies  $TP_1$  property if and only if :

$$op_1 \circ T(op_2, op_1) \equiv op_2 \circ T(op_1, op_2)$$

where  $op_i \circ op_j$  denotes the sequence of operations containing  $op_i$  followed by  $op_j$  ; and where  $\equiv$  denotes equivalence of the two sequences of operations.

This first property  $TP_1$  expresses equivalence between two sequences. Given two concurrent operations  $op_1$  and  $op_2$ , the execution of the sequence of  $op_1$  followed by  $T(op_2, op_1)$  on a state  $S$  must produce the same state as the execution of the sequence of  $op_2$  followed by  $T(op_1, op_2)$ .

**Definition 3** For every three concurrent operations  $op_1$ ,  $op_2$  and  $op_3$  defined on the same state, the transformation function  $T$  satisfies  $TP_2$  property if and only if:

$$T(op_3, op_1 \circ T(op_2, op_1)) = T(op_3, op_2 \circ T(op_1, op_2))$$

This second property  $TP_2$  stipulates equality between two operations transformed with regard to two equivalent<sup>2</sup> sequences of operations. Given three operations  $op_1$ ,  $op_2$  and

<sup>1</sup>In the literature, this function is also called forward transformation or inclusion transformation.

<sup>2</sup>These two sequences are equivalent by transformation.

$op_3$ , the transformation of  $op_3$  with regard to the sequence formed by  $op_2$  followed by  $T(op_1, op_2)$  must give the same operation as the transformation of  $op_3$  with regard to the sequence formed by  $op_1$  followed by  $T(op_2, op_1)$ .

Ressel et al. [23] demonstrated that these two properties  $TP_1$  and  $TP_2$  are sufficient to ensure convergence of copies independently of the order in which concurrent operations are transformed.

With a correct set of transformation functions, the integration algorithm ensures consistency and the resulting collaborative editing tools would be reliable. Indeed, most of the OT integration algorithms have been proven [25], [17] to ensure convergence of copies if the underlying transformation functions satisfy the properties. Unfortunately, currently none of the transformation functions proposed are correct regarding  $TP_1$  and  $TP_2$  properties. Imine et al. [11] used a formal approach based on a theorem prover to check correctness of all previously published set of transformation functions. And, they found counterexamples for each proposition. Li et al. [13] gave also a counter-example for the transformation functions proposed by Imine et al. [11]. In Section IV, we will give a counter-example for [13]. Therefore, currently there is no correct transformation functions and consequently the operational transformation cannot be used to build a safe decentralized collaborative editing system.

In this paper, we present the first set of transformation functions satisfying both  $TP_1$  and  $TP_2$  properties. Consequently, these transformation functions could be used with any integration algorithm previously published.

### III. TRANSFORMATION FUNCTIONS

All the transformation functions previously published do not satisfy  $TP_2$  property because they fail to solve a variant of the same problem. Even if some counterexamples are more complicated – involving partial concurrency between operations – the basic problem remains the same. This problem, presented in Figure 3, involves three concurrent operations :  $ins(2, x)$ ,  $del(2)$  and  $ins(3, y)$ .

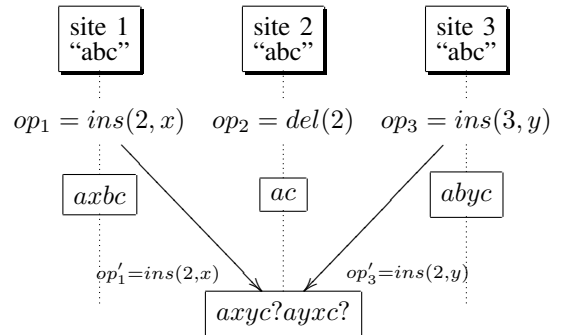


Fig. 3. Common problem.

When  $op_1 = ins(2, x)$  is received on site 2, it is transformed according to  $op_2$ . This transformation does not change insertion position of  $op_1$ , and, returns as result the operation  $op'_1 = ins(2, x)$ . When  $op_3 = ins(3, y)$  is received on site 2, it is transformed according to  $op_2$ . Its insertion position is

decreased and thus it becomes  $op'_3 = ins(2, y)$ . At this point on site 2, if the algorithm has to transform  $op'_1$  according to  $op'_3$ , or  $op'_3$  according to  $op'_1$ , then it must break a tie because their current positions of insertion are equal. To break this tie, all the approaches choose a different way: initial positions for IMOR [11], sets of concurrent deletions for Suleiman et al. [25] or state difference for Li et al. [13]. Unfortunately, all these approaches fail to order correctly  $x$  and  $y$  in some cases. And, all the counter-examples [11], [13] violating  $TP_2$  property are only instances of this tie. Nevertheless, on the initial state, character  $b$  obviously separates  $x$  and  $y$ ;  $b$  is called landmark character by Li et al. [15]. Consequently, the algorithm must insert  $x$  in front of  $y$  on all sites.

#### A. (U)ncompacted-TTF Model

Our idea is to keep the deleted character  $b$  as a tombstone. If a character is deleted, the system maintains useful information about its former position but not its whole content. Tombstones are well known in distributed systems. For example, they are heavily used in Usenet<sup>3</sup> to make conflicts between update/delete operations non ambiguous [24].

For a character string, it is equivalent to keep the character in its position and mark the character as invisible. If the shared data is a more complex linear data structures such as a list of lines instead of list of characters, it means that the system will maintain the identity of the line but not its content.

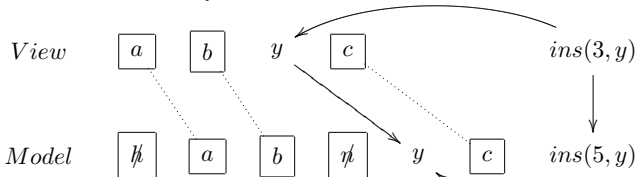


Fig. 4. Relation between view and model in TTF.

Consequently, hidden characters must remain present in the model of the string from a site, but are hidden from the view of the string seen by the user. The model of the string  $S$  becomes a sequence of ordered pairs (*character*, *visible*) where *visible* is a boolean attribute. This model and its behaviour are illustrated in Figure 4. Suppose a user inserts the character  $y$  between  $b$  and  $c$ , then the operation  $ins(3, y)$  is generated on the view, but this operation is executed on the model as  $ins(5, y)$  since the characters  $h$  and  $n$  have previously existed. This is also the operation  $ins(5, y)$  which is broadcast and will be transformed on all other sites.

To perform the conversion between the position of an operation in the model and the position in the view, the system uses the following function:

```

viewToModel(int pview, S) : int {
  int n=1, j=1;
  while ((j ≤ length(S)) and (n<pview or not S[j].visible)) {
    if (S[j].visible) n++;
    j++;
  }
  return j;
}

```

<sup>3</sup>Usenet is the system network in charge of replicating newsgroups.

Suppose the system has to execute a newly operation  $ins(p, c)$  generated from the view, then, on the model, it has to seek out the first position located after finding  $p$  visible characters and then skips all removed characters (tombstones) situated after this current position. The complete functions to execute local operations and remote operations are described in Figure 5 and in Figure 6. The function  $shiftRight(S, p)$  makes a new room in the string  $S$  at position  $p$  by shifting to right every character in the range  $[p; length(S)]$ . In the same way,  $shiftLeft(S, p)$  removes the element located at position  $p - 1$  in the string  $S$  by shifting to left every character in the range  $[p; length(S)]$ . It is worth to note these functions are commonly required to insert or remove an element in the string, even without using the TTF data model.

```

executeLocal(ins(x,pview), S) {
  int pmodel=viewToModel(pview);
  shiftRight(S,pmodel); // make room for the new character
  S[pmodel]=x;
  broadcast(ins(x,pmodel));
}
executeLocal(del(pview), S) {
  int pmodel=viewToModel(pview);
  S[pmodel].visible=false;
  broadcast(del(pmodel));
}

```

Fig. 5. Local executions in the U-TTF model.

Executing a received operation is different from executing a local operation. So we define two functions  $executeRemote(ins(x, p), S)$  and  $executeRemote(del(p), S)$  described in Figure 6.

```

executeRemote(ins(x,pmodel), S) {
  shiftRight(S,pmodel); // make room for the new character
  S[pmodel]=x;
}
executeRemote(del(pmodel), S) {
  S[pmodel].visible=false;
}

```

Fig. 6. Remote executions in the U-TTF model.

This strategy leads to the trivial transformation functions presented in Figure 7. An additional parameter  $sid_i$  has been added to operations. It identifies in a unique manner the site on which the operation has been generated. Identifiers of two insert operations are compared in order to break the tie when two insertions have been performed concurrently at the same position.

Since the execution of a  $del()$  operation replaces the removed character with a tombstone, it does not affect the position of other characters in the string. Consequently, performing a transformation of an operation  $op$  according to a  $del()$  operation does not modify the effect position of  $op$ .

Transformation functions of any operation according to an  $ins()$  operation are defined as originally done by Res-sel et al. [23]. When two insert operations have the same

```

T(ins(p1, c1, sid1), ins(p2, c2, sid2)) :-
  if (p1 < p2) return ins(p1, c1, sid1)
  else if (p1 = p2 and sid1 < sid2) return ins(p1, c1, sid1)
  else return ins(p1 + 1, c1, sid1)

T(del(p1, sid1), ins(p2, c2, sid2)) :-
  if (p1 < p2) return del(p1, sid1)
  else return del(p1 + 1, sid1)

T(ins(p1, c1, sid1), del(p2, sid2)) :-
  return ins(p1, c1, sid1)

T(del(p1, sid1), del(p2, sid2)) :-
  return del(p1, sid1)

```

Fig. 7. TTF transformation functions.

position  $p$ , the character produced by the site with the lower range is inserted at  $p$ ; the other character is inserted at position  $p + 1$ .

We have proven that TTF ensure  $TP_1$  and  $TP_2$ . Due to space limitations, the proof is not included in this paper. This proof was built using the automatic theorem prover SPIKE [3]. The full specification and how the proof is built by the theorem prover is available in [19]. But even without SPIKE, it is easy to see that, in opposite to traditional transformation functions, the TTF are monotonic transformations of the effect position of operations since they only compute additions. Hence, the position of one character will grow monotonically to the same value independently of the equivalent transformation path taken.

This monotonic property has another interesting consequence: TTF preserve order relationships between characters which is considered in [13] as an instantiation of the intention preservation criterion defined by Sun et al [28]. In [13], the intention of  $ins(p, c)$  operation is expressed by the relation  $\prec$ . If one user generates  $op = ins(p, c)$  on a site where  $x$  is visible at a position less than  $p$ , and  $y$  is visible at a position greater or equals than  $p$ , then the ordering  $x \prec c \prec y$  is set. Concerning a  $del(p)$  operation executed on a string  $S$ , its intention is to remove the character  $S[p]$ . Please note that in our model the  $del(p)$  does not remove the character but makes it invisible. Preserving intention of  $ins()$  operations means that the  $\prec$  relations hold on all further states. Since in our model  $del()$  operations do not affect the positions of characters, this ordering will always be kept sound on the generation site of  $op$ . Moreover, using our TTF all copies on all sites will eventually converge to the same string. Consequently, for any operation  $ins(p, c)$  the ordering  $x \prec c \prec y$  will be preserved in any further states. In other words, TTF can ensure intention preservation criterion as defined in [13].

### B. Inverses of Transformation Functions

Some algorithms such as SOCT2 [25], GOT [28] and GOTO [27] required to define additional functions called “exclusion transformation” [28]. These functions are the inverses of the transformation functions. Since the TTF transformation

functions are injective functions, defining their inverse functions is straightforward. Figure 8 gives the definitions of the inverses of the TTF transformation functions.

```

T-1(ins(p1, c1, sid1), ins(p2, c2, sid2)) :-
  if (p1 < p2) return ins(p1, c1, sid1)
  else if (p1 = p2 and sid1 < sid2) return ins(p1, c1, sid1)
  else return ins(p1 - 1, c1, sid1)

T-1(del(p1, sid1), ins(p2, c2, sid2)) :-
  if (p1 < p2) return del(p1, sid1)
  else return del(p1 - 1, sid1)

T-1(ins(p1, c1, sid1), del(p2, sid2)) :-
  return ins(p1, c1, sid1)

T-1(del(p1, sid1), del(p2, sid2)) :-
  return del(p1, sid1)

```

Fig. 8. Inverses of TTF transformation functions.

The preconditions of the inverse of a transformation function required that the first parameter operation is resulting from a transformation according to the second one. Indeed, it is not allowed to call this inverse function in order to swap two operations causally dependent because these operations were not concurrent, and thus were not previously “serialised” using transformation functions. For instance, consider the string of characters “abc” and two operations  $op_1 = ins(2, x)$  and  $op_2 = del(2)$  executed in this order on the string of characters. The resulting state after the executions of  $op_1$  and  $op_2$  is “abc”. Since, these two operations are causally dependent as  $op_2$  deletes the character inserted by  $op_1$ , it is forbidden to try to swap these two operations in order to execute them in the reverse order ( $op_2$  then  $op_1$ ). Performing such transformations will compute the sequence of operations  $[del(1), ins(1, x)]$  whose execution will lead to the wrong state “xbc”. These preconditions are always ensured by OT integration algorithms such as SOCT2 for example.

### C. Optimising TTF Approach

Compared to existing approach, the function `viewToModel()` has to be computed each time a character is generated. Its time complexity is linear to the size of the text. In case of asynchronous editing this complexity has no impact since operations are generated when user edition is finished.

In case of real time editing, this extra time will slow down the local response time if the text is long. However, we can obtain a good response time by a simple optimisation. In an editor, local operations are relative to the caret position. Indeed, the `viewToModel()` computes the position of the caret in the model. This position could be stored as the global variable `caretPosition`. And, its value is updated when the user moves the caret in the text with the procedure `updateCaret(oldPos, newPos, S)`. The parameters `oldPos` and `newPos` are the old and new caret positions in the view presented to the user.

```

updateCaret(int oldPos, int newPos, S) {
  int i = oldPos;
  if (newPos ≥ oldPos) { // move right
    while (i <> newPos) {
      if (S[caretPosition].visible) i++;
      caretPosition++;
    }
    // skip removed char
    while (caretPosition ≤ length(S)
      and not S[caretPosition].visible)
      caretPosition++;
  } else { // move left
    while (i <> newPos) {
      if (S[caretPosition-1].visible) i--;
      caretPosition--;
    }
  }
}

```

The `executeLocal()` functions could be rewritten as follows:

```

executeLocal(ins(x,caretPosition), S) {
  shiftRight(S, caretPosition);
  S[caretPosition] = x;
  broadcast(ins(x, caretPosition));
}
executeLocal(del(caretPosition), S) {
  S[caretPosition].visible = false;
  broadcast(del(caretPosition));
}

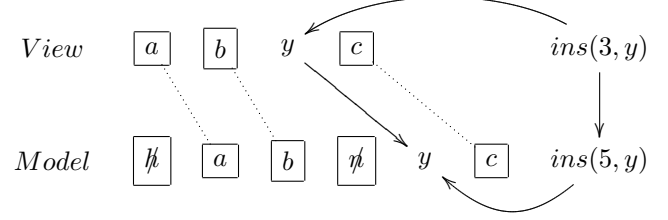
```

#### D. (D)elta-TTF Model

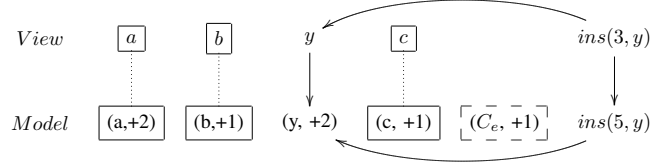
Although tombstones is a very easy solution for the OT approach, TTF retains a tombstone to mark a deleted character. So, the space overhead of tombstones grows indefinitely. Two solutions are generally used to solve this issue in distributed systems. The first one is based on an expiration period which is associated to each tombstone. A tombstone is definitively suppressed as soon as its period expires. It is well-known that this method is unsafe. An operation assuming presence of some tombstones might arrive after their expiration periods, in this case this operation cannot be executed properly. The second solution employs a two-phase protocol to purge safely the tombstones as described in [24]. Unfortunately, such a protocol requires all sites must be alive for the algorithm to make progress. In other words, it means that all participants using the collaborative editing system must be and stay connected until the garbageing is finished. This assumption is not suitable for asynchronous collaborative editing systems.

In our context, this problem is equivalent to managing a sparse array where deleted characters are considered as zero entries. The basic idea when storing this kind of array is to only store the non-zero entries as opposed to storing all entries. Hence, we present now another model to store the local string. Each visible character keeps an integer value equals to 1 + the number of invisible characters located between it and the visible character preceding it. This new model is depicted in Figure 9(b). This new model requires to add a special invisible character named  $C_e$  which is used to keep the number of

characters plus one which were located after the last visible characters and have been removed.



(a) TTF uncompact model



(b) TTF delta model

Fig. 9. Difference between U-TTF and D-TTF models.

The `viewToModelID` function is rewritten as follows:

```

viewToModelID(int pos, S) : int {
  return  $\sum_{k=1}^{pos} S[k].offset$ ;
}

```

The functions of Figure 10 describe the execution of a local insert or delete operation.  $S$  is a sequence of pair (*character, relative position*).  $S[p].offset$  returns the relative position of the character stored at position  $p$  in  $S$ . Figure 11 defines how to execute a received operation.

```

executeLocal(ins(x,p), S) {
  >>>> CONFLICT >>>> ADD – ADD
  > shiftRight(S, p); // make room for the new character
  ==== CH2 / momo54
  > shiftRight(S, p);
  <<<< CONFLICT <<<<
  S[p] = (x, S[p+1].offset);
  S[p+1].offset = 1;
  broadcast(ins(x, viewToModelID(p, S)));
}
executeLocal(del(p), S) {
  S[p+1].offset = S[p+1].offset + S[p].offset;
  >>>> CONFLICT >>>> ADD – ADD
  >> int pos = viewToModelID(p, S);
  ==== CH2 / momo54
  >>>>> CONFLICT >>>>> ADD – ADD
  >> pos = viewToModelID(p, S);
  >==== CH2 / momo54
  >> shiftLeft(S, p+1);
  <<<<< CONFLICT <<<<<
  <<<<< CONFLICT <<<<<
  broadcast(del(pos));
}

```

Fig. 10. Local executions in the D-TTF model.

```

executeRemote(ins(x,p), S) {
  int sum = 0, i = 0;
  while (i ≤ length(S) and sum < p) {
    i++;
    sum += S[i].offset;
  }
  shiftRight(S, i);
  S[i] = (x, p-(sum-S[i+1].offset));
  S[i+1].offset = sum-p+1;
}
executeRemote(del(p), S) {
  int sum = 0, i = 0;
  while (i ≤ length(S) and sum < p) {
    i++;
    sum += S[i].offset;
  }
  if (sum==p) {
    S[i+1].offset = S[i+1].offset + S[i].offset;
    shiftLeft(S,i+1);
  }
}

```

Fig. 11. Remote executions in the D-TTF model.

Theoretical complexity of every operation execution is  $O(n)$ . However, we can apply the same optimization we use for the uncompact model. The position of the caret in the model is stored and updated when user moves.

#### IV. RELATED WORK

During the last decades, the operational transformation model has triggered a growing enthusiasm for maintaining consistency in collaborative editing systems. Since the initial work from Ellis et al. on dOPT [6] algorithm was found false regarding the  $TP_2$  properties, two ways of research are explored.

The idea of the first approach is to avoid the need of  $TP_2$  property. There are mainly two works based on this approach: the GOT [28] algorithm and the SOCT4 [29] algorithm. In GOT, all operations will be eventually executed in the same total order on every site. In this manner, convergence of copies is ensured even if transformation functions do not satisfy  $TP_1$  nor  $TP_2$  property. These two properties are not required because on every site each operation will be transformed according to concurrent operations in the same order. This algorithm has one drawback. Since in OT approach, local operations are always executed immediately on the local copy, some local operations could be executed before the arrival of some remote operations. However, the executions of these remote operations might precede regarding the total order the execution of the local operations. To solve this issue, GOT uses an undo-do-redo mechanism [28] for undoing the local operations, then executes the remote operations and finally re-executes the undone operations. This undo-do-redo mechanism requires the exclusion transformation functions which of course must satisfy the reversibility property. Unfortunately, until now there is no transformation satisfying this property. The SOCT4 algorithm relies also on a total order, but in place

of the undo-do-redo mechanism, it restricts the broadcast of local operations. Indeed, in SOCT4, a site can send its local operations only if it has integrated all the preceding remote operations regarding the total order. This algorithm requires only the  $TP_1$  property to be satisfied on transformation functions to ensure convergence. Unfortunately, the continuous total order required by SOCT4 is generally implemented using a central time-stamper which restricts considerably the collaboration interaction practices. Moreover, if there exist transformation functions satisfying  $TP_1$  condition, they do not achieve intention preservation as defined in [13]. Hence, all copies will eventually converge to a unique state that might violate this definition of intentions of operations. The TTF transformation functions could be used to fix correctness issues of current collaborative systems based on GOT or SOCT4 algorithms.

The purpose of the second approach is to find transformation functions satisfying  $TP_2$  property. A lot of integration algorithms, such as adOPTed [23], SOCT2 [25] or GOTO [27], were proposed assuming that transformation functions will satisfy the  $TP_2$  property. Each algorithm was proposed in association with some transformation functions. None of these transformation functions are correct regarding  $TP_2$  properties as summarised in [11]. Authors of [11] have proposed some transformation functions, but Li et al [13] have found a counter-example for them violating  $TP_2$  and intention preservation.

It exists a  $TP_2$  counter-example for all existing transformation functions except the SDT transformation functions. SDT [13] stands for State-Difference Transformations. Authors wrote that SDT ensures  $TP_1$  and  $TP_2$ . Unfortunately, we found a counter-example with the help of the SPIKE theorem prover [19].

When the system has to transform one operation  $op_1$  according to a second concurrent operation  $op_2$ , it performs the following steps. First, it identifies the last common state (LSP) from which both concurrent operations were executed. Then, it computes the sequence  $SD$  of operations that could be executed on LSP to lead to the state on which  $op_2$  is defined. Next, it computes  $\beta(op_1)$  the potential effect position of  $op_1$  on LSP in excluding  $SD$  from  $op_1$ . It performs the same with  $op_2$  to get  $\beta(op_2)$ . Finally, these two positions are compared in the transformation for inferring the shift of the effect position of  $op_1$ .

The Figure 12 describes the SDT counter-example. The assumption  $\beta(op_{41}) = \beta(op_2) = \beta(op_3)$  could be satisfied if  $op_3$  is generated on a state preceding the definition state of  $op_2$  and  $op_3$ . Since,  $\beta(op_3) = \beta(op_{41})$ , SDT functions compared the effect positions to break the tie between the two concurrent operations  $op_3$  and  $op_{41}$ . On site 1, the positions are different  $p(op_{41}) < p(op_3)$  thus positions determine the result of the transformations. But, on site 2, the positions are equals  $p(op_{41}) = p(op_3)$ , and then the site priority is used to break the tie. Since on two sites, two different mechanisms are used to break the tie, a tricky choice in site priorities will lead to the divergence illustrated in Figure 12. We also

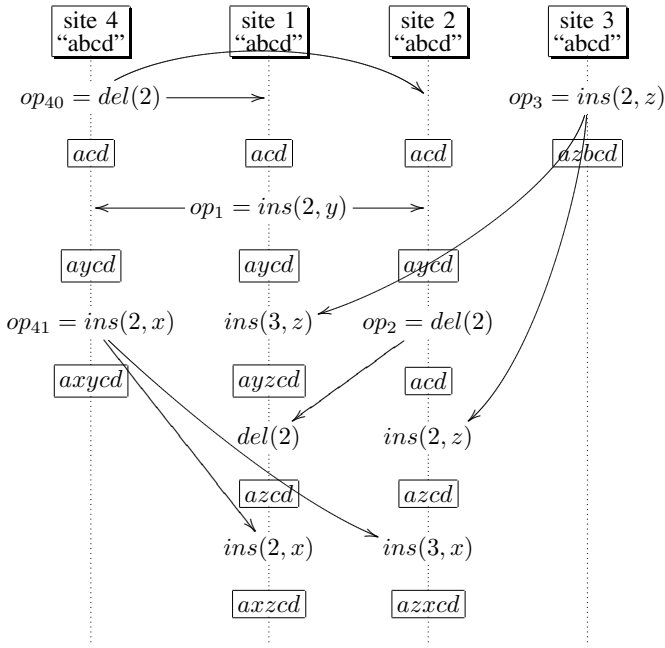


Fig. 12. SDT counter-example

verified the proof published [14]. We found that the proof of Lemma 5 is incomplete. Indeed, in this proof, authors consider two cases:  $\overline{O_3} \rightarrow (O_1 \parallel O_2)$  or  $\overline{O_3} \parallel (O_1 \parallel O_2)$ . Unfortunately, the proof of Lemma 5 does not consider the case where  $\overline{O_3} \rightarrow O_1$  and  $(\overline{O_3} \parallel O_2)$  and  $(O_1 \parallel O_2)$ . The counter-example presented in Figure 12 instantiates this latter case with:  $O_3 = op_2 = del(2)$ , so  $\overline{O_3} = op_1 = ins(2, y)$ ;  $O_2 = op_3 = ins(2, z)$ ;  $O_1 = op_{41} = ins(2, x)$ .

Consequently, it means that, currently, only TTF can be used to instantiate the Ressel’s framework. Other research proposals consider that the solution is not to instantiate the Ressel’s framework but to propose a new framework.

ABT [16] is an alternative approach to address consistency problems of existing OT algorithms. The first motivation of this work was the difficulty to find transformation functions that verify  $TP_1$  and  $TP_2$ . We demonstrate in this paper  $TP_1$  and  $TP_2$  can be easily achieved by simple transformation functions. The second motivation is that intentions of operations must be preserved during transformations, but they were not defined in the Ressel’s model and were not formalized when defined in [28]. Anyway, we proved in Section III-A, TTF preserve order relationships between characters. These relationships correspond to the effect relations as defined in ABT. Hence, we presented a set of transformation functions that verify all requirements described in [16] while keeping the Ressel’s model and related integration algorithms. However, the ABT model is defined only for linear structures while the Ressel’s model is independent of structures of shared data.

There is another approach which does not transform operations: the Mark & Retrace technique [9]. When a remote operation has to be integrated, the document’s address space

is retraced to the state at the time the operation was generated. This *retracing* is done by marking effective the characters which were present at the generation time – even the ones which have been deleted afterwards –, and ineffective the characters which are inserted concurrently. Then, in the case of an insertion, the new character has to be inserted between its previous and next character. But, since some concurrent inserted and deleted characters might be present between its previous and next character, a *range-scan* function is applied to order totally these characters. These functions and their proofs of correctness clearly depend on the linear structure of the shared object. Thus, applying this technique to other types of structure will require designing new algorithms and proving them. Moreover, there is no easy optimisation to perform in order to garbage operations or deleted characters. Furthermore, when working with some particular shared object; it is sometimes impossible to achieve convergence without cancelling operations except by allowing transforming them.

The WOOT framework [20] works in a quite similar way to the *range-scan* function of Mark & Retrace, except that it does not require the execution of the retracing process. The previous and the next characters between which an operation is performed are found in the string using their unique identifiers. This feature leverages the requirement of state vectors since they might be a weak point in large-scale systems. Unfortunately, the weakness concerning the impossibility to garbage characters and operations remains. And, this framework is not generic since it relies strongly on some structural precedence relation on characters.

## V. CONCLUSIONS AND FUTURE WORK

In this paper, we proposed the Tombstone Transformation Functions for maintaining consistency in collaborative editing systems based on the operational transformation approach. These functions satisfy the correctness properties  $TP_1$  and  $TP_2$ , and consequently they are able to preserve intentions while ensuring copies convergence. These functions can be used with existing integration algorithms such as adOPTed [23], SOCT2 [25] or SOCT4 [29] and therefore could be easily used to replace the wrong transformations used in existing systems.

In this paper we presented our solution for linear structures, but the simplicity of the mechanism offers support for the extension to other data structures.

In collaborative editing systems, the ability to undo operations [21] is a widely used feature. In the operational transformation community, several propositions [23], [22], [26], [7] have already been made. It is worth to point out that maintaining tombstones greatly simplify the design of an undo mechanism. Indeed, since all the locations of deleted elements are kept, it is easy to restore them at their right place. One of the main differences with the previous approaches is that undoing the deletion of an element is different from inserting this element. This means a new operation has to be added in the model. We are currently working [30] on the



transposition to our TTF model of the correctness properties for undo mechanism defined in [7].

As future work, we also plan to extend our TTF to string-wise operations as described in [28], and hierarchical data structures as presented in [4], [10].

Our aim is to deploy a collaborative editing system on a peer-to-peer network. Actually, most of the OT algorithms that do not require a central site are based on version vectors to times updates. Unfortunately, version vectors do not scale and cannot be used in a system with a large number of sites. We are investigating ways of extending operational transformation algorithms to fit the requirements of this kind of network.

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