On-the-fly Analysis
of Distributed Computations

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May 13, 1994

Abstract
At some abstraction level a distributed computation can be modelled as a partial
order on a set of observable events. This paper presents an analysis technique
which can be superimposed on distributed computations to analyze control flows
terminating at observable events. A general algorithm working on the longest
control flows of distributed computations is introduced. Moreover it is shown how
this algorithm can be simplified according to the definition of observable events or
to the set of control flows we want to analyze.

Key Words: Distributed computation, observable event, longest control flows,
causal precedence, sequences analysis.

1 Introduction
Since Lamport’s seminal paper [8], distributed computations are modelled as sets of
events structured by partial order relations. For a particular computation, events pro-
duced by each process are totally ordered and communications creates dependencies
among events belonging to distinct processes. These partial order relations on events
are generally called happened before (with respect to a logical time frame) or causal
precedence. They formally express control flows and their mutual dependencies which
organize distributed executions.

One important topic addressed by computer science is the analysis of sequences of
symbols or words (e.g. syntactical analysis, pattern recognition, etc). Well-suited for-
malizations have been designed and specialized tools have been implemented to make
feasible such analyses in specific domains (formal languages and automata theory are the most famous example of these works). In this paper we are interested in analyzing on the fly the set of “words” produced by a distributed computation; a word being defined from sequences of relevant events produced by an execution of a distributed program. The practical motivation of our work comes from debugging, testing and monitoring of distributed computations [5, 7]. In this context we choose to explore analysis techniques of distributed computations which must be done on-the-fly and without delay (this is particularly important in the context of reactive monitoring). These constraints eliminate the possibility to log events produced by each process (as the analysis cannot be done off-line) or to use an additional process (monitor) that would receive notification messages sent by processes of the computation in order to analyze their traces (as in that case notification messages would add some delay between event occurrences and their knowledge by the monitor). In other words we constraint our analysis mechanism to be superimposed on the computation and to use only a piggybacking technique to convey analysis related informations from one process to another.

According to the aim of the analysis (detection of a property, for example) only a subset of all the events generated by a distributed execution are meaningful to the user, these events are called observable events. From this point of view, the other events are ignored at the abstraction level considered; they participate only in the establishment of causal dependencies between observable events. Sequences of observable events are defined by the partial order relation associated with the computation. Although each observable event of the computation is unique, several events can be execution occurrences of the same action for example. So a labelling function is introduced and the sequences of observable events are associated with words (concatenation of labels). The analysis is then carried out on the fly and without delay on these words. The analysis considered in this paper is based on finite state automata. Other kinds of automata could be used but finite state automata are sufficient to illustrate our analysis technique; moreover finite state automata can solve interesting practical problems (see Section 5) and allow an efficient analysis, as far as the automaton is concerned, as they require only the piggybacking of a bounded number of bits –one per state of the automaton– to do the analysis.

The paper is divided into 4 main sections. Section 2 presents the model of distributed computation; Section 3 presents the kind of analysis we are interested in, the definition of languages (set of words) associated with distributed computations and the two question (satisfaction rules) which can be answered by the analysis. Section 4 presents a general distributed algorithm which, in this context, analyzes on the fly and without delay a distributed computation. Section 5 examines particular cases according to the position of observable events with respect to communication events.
2 Distributed Computations

2.1 Distributed programs

We are interested here by distributed computations. Such computations result from the execution of distributed programs. A distributed program is made of $n$ sequential processes $P_1, \ldots, P_n$ which synchronize and communicate by the only means of message passing. A distributed program can be directly produced by a programmer or can be the result of the compilation of a parallel or sequential program for a distributed memory parallel machine. Processes that realize the distributed computation execute actions which are either communication actions (sending of a message, reception of a message) or internal actions (all the other actions). Execution of an action is called an event.

2.2 Model of distributed computations

2.2.1 Lamport’s precedence relation

When executed, each sequential process $P_i$ produces a set of events $E_i$ totally ordered by a local precedence relation $<$. This set $E_i$ can be partitioned into two subsets:

- $I_i$: the set of internal events of $P_i$ (resulting from internal actions);
- $X_i$: the set of communication events of $P_i$ (send and receive events).

The set $E = \bigcup_i E_i$ of all the events produced by the distributed execution is partially ordered by Lamport’s relation called happened before or causal precedence [8]. The resulting poset is noted $\hat{E} = (E, \leq_e)$:

$$\forall x \in E_i, y \in E_j : x \leq_e y \overset{\text{def}}{=} \begin{cases} x = y \\
\text{or} \\
i = j \text{ and } x <_i y \\
\text{or} \\
x \text{ is the sending of a message and } y \text{ its reception} \\
\text{or} \\
\exists z \text{ such that } x \leq_e z \text{ and } z \leq_e y
\end{cases}$$

2.2.2 Abstraction level and observable events

Analysis of a (distributed) computation is always done at some abstraction level (usually language, system or hardware level). For an abstraction level, a distributed computation is characterized by the events that must be observed in order to analyze it. So we consider here that, for a given abstraction level, only a subset of internal events are relevant; these events are called observable events and result from execution of specific actions (for example, modifications of some processes variables); these actions will be identified by labels as described in Section 3.2. Communication events create causal dependencies.
between observable events but are not supposed to be observable (if necessary a communication event can be made observable by generating an additional internal event just before it—in case of a send—or just after it—in case of a receive--; see Section 5.2).

So an abstraction level is a screen that filters out all irrelevant events and keeps all causal dependencies between relevant events. Let \( O_i \subseteq I_i \) be the set of observable events of \( P_i \) and \( O = \bigcup O_i \). At the abstraction level considered the distributed computation is characterized by the poset \( \hat{O} = (O, \leq_o) \) with \( \leq_o \) defined by\(^1\):

\[
\forall x, y \in O : x \leq_o y \iff x \leq E y
\]

Figure 1 displays a distributed computation in the classical space-time diagram. Observable events are denoted by black points; exchanges of messages are represented by arrows going from one process line to another one.

![Figure 1: A distributed computation.](image)

Figure 2 displays the poset \( \hat{O} = (O, \leq_o) \) associated with the distributed computation of Figure 1 (only non reflexive and non-transitive edges are represented).

![Figure 2: A poset \( \hat{O} = (O, \leq_o) \)](image)

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\(^1\)We use \( x <_o y \) as a shorthand for \( x \leq_o y \ and \ x \neq y.\)
2.3 Deciding about order of events

By using a vector clock mechanism [2, 9] a timestamp can be associated with each event and used to decide about order of events. Such a vector timestamp $V_x[1..n]$ associated with event $x$ is such that:

$$\forall k \in 1..n : V_x[k] = \lvert \{ z \in O_k : z \leq_O x \} \rvert$$

and we have

$$\forall x \in O_i, y \in O : x \leq_O y \iff V_x[i] \leq V_y[i]$$

These timestamps are obtained by using the classical vector clock mechanism [2, 9]:

- each process $P_i$ maintains an integer vector $V_i[1..n]$ initialised to 0;
- each time $P_i$ produces an observable event $x \in O_i$, it executes: $V_i[i] := V_i[i] + 1; V_x := V_i$;
- each time $P_i$ sends a message, it adds $V_i$ to the message;
- each time $P_i$ receives a message from $P_j$, piggybacking $V_j$ it executes: $\forall x \in 1..n : V_i[k] := max(V_i[k], V_j[k])$.

3 On the Fly Analysis of a Distributed Computation

3.1 Covering graph of causal dependencies

A way to perform analyses of distributed computations on the fly and without delay is to carry them out incrementally each time an observable event is produced. So the analysis done when such an event $x$ is produced is on the set of its causal predecessors, denoted $\text{pred } x$:

$$\text{pred } x = \{ y \in O : y \leq_O x \}$$

Among all the causal predecessors of $x$ some are its immediate predecessors. If it exists the immediate predecessor of $x$ on some process $P_j$ is unique. It constitutes the singleton or empty set $\text{im_pred}_j x$. Formally:

$$(\text{im_pred}_j x = \{ y \}) \equiv \begin{cases} y <_O x \\ \text{and} \\ y \in O_j \\ \text{and} \\ \forall z \in O : (y <_O z \text{ and } z <_O x) \end{cases}$$
Figure 3: $b$ is an immediate predecessor of $c$.

For example, in Figure 3, the observable event $c$ has an immediate predecessor on $P_j$ ($im\_pred_j c = \{b\}$) and there is no immediate predecessor of $x$ on process $P_i$ ($im\_pred_i c = \{}$).

Let $\hat{C}$ be the covering graph of $\hat{O}$ (i.e. $\hat{C}$ is $\hat{O}$ from which all transitivity edges have been suppressed); we have:

$$(y, x) \in \hat{C} \iff \exists j \in 1..n : im\_pred_j x = \{y\}$$

Considering an observable event $x$, the analysis is carried out on the paths of $\hat{C}$ beginning with an observable event without predecessor and ending at $x$; let $C(x)$ be the set of all these paths. In Figure 3 we have $C(c) = \{abc, aefc, defc\}$. Paths of $C(x)$ include all events of $pred x$ and correspond to the “longest control flows” that are needed to produce $x$. We can see that, in the previous example, the path $ac$ is not a “longest control flow”.

It would be possible to consider a set of paths ending at $x$ larger than $C(x)$ to perform the analysis. We choose the previous set $C(x)$ essentially for the generality of the associated analysis algorithm described in Section 4. The resulting algorithm is very general and can be simplified to work on a larger set of control flows ending at some observable event $x$. Such an algorithm is sketched at Section 5.3. When considering the observable event $c$ in Figure 3, this simplified algorithm works on the set of paths: \{abc, ac, aec, aefc, dec, defc\} which represents all possible control flows ending at $c$ and not only the “longest” ones.

### 3.2 Language associated with observable events

Observable events are execution of relevant actions at the abstraction level considered. These actions define an alphabet $A$ and a labelling function $\lambda: O \rightarrow A$ associates with each observable event an element of the alphabet $A$.

A language $\mathcal{L}(x)$ (set of words of $A^*$) is associated with each observable event $x$ in the following way:

$$\mathcal{L}(x) = \{\lambda(\sigma) : \sigma \in C(x)\}$$

where $\lambda(x_1x_2\ldots x_k) = \lambda(x_1)\lambda(x_2)\cdots \lambda(x_k)$. 

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3.3 Specifying a pattern

The analysis consists in answering whether the computation meets some pattern. The patterns we are here interested in are described with a finite state automaton $\Phi = (A, Q, q_0, \delta, F)$ where:

- $A$ is the alphabet (labels of observable events);
- $Q$ is the set of states;
- $q_0$ is the initial state;
- $\delta \subseteq Q \times A \times Q$ is the transition function;
- $F \subseteq Q$ is the set of final states.

Other kinds of automata could be used to specify patterns. But as claimed in the introduction, finite state automata are sufficient to describe a lot of practical properties (whose linked predicates [4] and atomic sequences of predicates [6] are special cases).

$L(\Phi)$ will denote the language recognized $\Phi$.

3.4 Satisfaction rules

Consider an observable event $x$. Two kinds of question can be answered according to kind of analysis we are interested in: either only one or all paths of $C(x)$ match the pattern described by the automaton $\Phi$. More formally the two following satisfaction rules are defined for $x \in O$:

$$x \models_3 \Phi \iff L(x) \cap L(\Phi) \neq \{\}$$
$$x \models_\forall \Phi \iff L(x) \subseteq L(\Phi)$$

The first satisfaction rules is useful in the context of debugging (with $\Phi$ describing a pattern revealing an error). The second one is more interesting in the field of on-line testing to verify an execution did not exhibit an anomalous behavior.

4 An On the Fly Analysis Algorithm

4.1 An incremental analysis

Consider an observable event $x$; let $\Phi(x)$ be the set of states reached by the automaton by analyzing all words of $L(x)$:

$$\Phi(x) = \{\delta^*(q_0, \sigma) : \sigma \in L(x)\}$$
where $\delta^*$ is the transition function of $\Phi$ extended to words. We have:

\[
x \models_{=\exists} \Phi \iff \Phi(x) \cap F \neq \emptyset
\]
\[
x \models_{=\forall} \Phi \iff \Phi(x) \subseteq F
\]

As the analysis is on $C(x)$, the set of “longest control flows” ending at $x$, $\Phi(x)$ can be computed in an incremental way by using the immediate predecessors of $x$:

\[
\Phi(x) = \left\{ \delta(q, \lambda(x)), \forall q \in \bigcup_j \Phi(\text{im}_{\text{pred}_{j}} x) \right\}
\]

with $\Phi(\emptyset) = \emptyset$ and $\Phi(\text{im}_{\text{pred}_{j}} x) = \{q_0\}$ if $x$ is a minimum of $O$.

### 4.2 Description of the algorithm

Each process $P_i$ is endowed with a vector clock $V_i[1..n]$ managed as described in Section 2.3. It is also associated with an array $\Phi_{\text{pred}_i}[1..n]$ of sets of elements of $Q$.

$\Phi_{\text{pred}_i}[j]$ is managed in such a way that supposing the next event $x$ produced by $P_i$ is an observable event, we have: $\Phi_{\text{pred}_i}[j] = \Phi(\text{im}_{\text{pred}_{j}} x)$ (the proof in Section 4.3 will establish this invariant).

The algorithm is defined by the 4 following statements S1 to S4 executed by each process $P_i$.

**S1:** initialization:

$\forall j \in 1..n : V_i[j] := 0$;

$\forall j \in 1..n, \Phi_{\text{pred}_i}[j] := \{q_0\}$;

**S2:** When $P_i$ produces an observable event $x$:

$V_i[i] := V_i[i] + 1$;

$\Phi_{\text{pred}_i}[i] := \{\delta(q, \lambda(x)), \forall q \in \bigcup_k \Phi_{\text{pred}_i}[k]\}$;

$\forall j \in 1..n, j \neq i : \Phi_{\text{pred}_i}[j] := \{\}$;

$\% \Phi(x) = \Phi_{\text{pred}_i}[i] \%$

**S3:** When $P_i$ sends a message to $P_k$:

$V_i[1..n]$ and $\Phi_{\text{pred}_i}[1..n]$ are added to the message;
S4: When $P_i$ receives from $P_k$ a message piggybacking $V_k$ and $\Phi_{pred_k}$:

\[\forall j \in 1..n : \text{do}\]

\[\text{case}\]

\[V_i[j] < V_k[j] \text{ then } \Phi_{pred_i}[j] := \Phi_{pred_k}[j]; \]
\[V_i[j] := V_k[j]\]

\[V_i[j] > V_k[j] \text{ then skip}\]

\[V_i[j] = V_k[j] \text{ then if } \Phi_{pred_i}[j] \neq \{\}\]
\[\text{then } \Phi_{pred_i}[j] := \Phi_{pred_k}[j]\]

\[\text{fi}\]

\[\text{end-case}\]

S2 acts as a reset: when it produces an observable event $x$, $P_i$ computes $\Phi(x)$ according to the states in which the automaton was when the immediate predecessors of $x$ were produced. Moreover if the next event produced by $P_i$ is observable it will have $x$ as the only immediate predecessor.

S4 updates the array $\Phi_{pred_i}$ in order, $\forall j$, the invariant relying the concrete variable $\Phi_{pred_i}[j]$ and the abstract variables $\Phi(im_{pred_j} x)$ be maintained. Vector clocks play an essential role in this management by permitting to know which events are immediate predecessors of each observable event of the distributed computation. Section 4.3 proves the correctness of this updating strategy.

4.3 Proof

Notation:

- For the sake of simplicity we will note $\pi(x) = i$ if $x \in E_i$, $\forall i \in 1..n$.

- In the following $\Phi_{pred}[k](t)$ denotes the value of $\Phi_{pred_{\pi(t)}[k]}$ just after the event $t$ has been produced.

- We assume that each process $P_i$ begins with an initial fictitious internal event $\bot_i \notin O$ such that $\forall j, \Phi_{pred}[j](\bot_i) = \{q_0\}$ ($\bot_i$ represents the initialization of $P_i$).

As explained in Section 4.2, we have to show the following proposition in order to prove the correctness of the algorithm (illustrated in Figure 4).

**Proposition**

Let $x \in O$ and $t \in E$ such that $t <_{\pi(x)} x$ and $\exists y \in E : t <_{\pi(y)} y \land y <_{\pi(x)} x$ then we have:

\[\forall j \in 1..n : \Phi_{pred}[j](t) = \Phi(im_{pred_j} x)\]
$\forall t \in E$, let $\text{last}_\omega(t)$ denote the event such that:

\[(\text{last}_\omega(t) <_{\pi(t)} t)\]

and

\[\text{last}_\omega(t) \in O \text{ or last}_\omega(t) \text{ is a receive event}\]

and

$\exists y$ such that $(y \in O$ or $y$ is a receive event$)$ and $\text{last}_\omega(t) <_{\pi(t)} y$ and $y <_{\pi(t)} t$ or such that

$\text{last}_\omega(t) = \perp_{\pi(t)}$ if there is neither receive nor an observable event on $P_{\pi(t)}$ before $t$.

Remark that if $t$ is an internal event or a send event, the algorithm does not change the array $\Phi_{\text{pred}, \pi(t)}[1..n]$. So, in order to proof the proposition, it is sufficient to prove that (see Figure 5):

\[\forall x \in O, \forall j \in 1..n : \Phi_{\text{pred}}[j](\text{last}_\omega(t)(x)) = \Phi(\text{im}_{\text{pred}} j x)\]

The proof is on the rank of $x$. More formally, let $\mathcal{M}^n$ and $\mathcal{O}^n$ be subsets of $O$ such that:

\[\mathcal{M}^0 = \{x \in O : \exists y \in O : y <_{\omega} x\} \quad \mathcal{O}^0 = O\]
\[\mathcal{M}^n = \{x \in \mathcal{O}^{n-1} : \exists y \in \mathcal{O}^{n-1} : y <_{\omega} x\} \quad \mathcal{O}^n = \mathcal{O}^{n-1} \setminus \mathcal{M}^{n-1}\]

With these notations we have to prove the following:

\[\forall r, \forall x \in \mathcal{M}^r, \forall j \in 1..n : \Phi_{\text{pred}}[j](\text{last}_\omega(t)(x)) = \Phi(\text{im}_{\text{pred}} j x)\]

Moreover we suppose there is no infinite chains $CC$ of $O$, such that $\exists x \in \mathcal{M}^n, y \in \mathcal{M}^{n+1} : x = \min(CC), y = \max(CC)$. This states that, for the distributed computations we consider, there is no an infinity of events between two events (this hypothesis will be called finiteness hypothesis in the following).

Figure 4: The proposition.
Base case

Let \( x \in \mathcal{M}^0 \). \( x \) is a minimum of \( O \), so \( \forall j \in 1..n : \Phi(\text{im} \text{pred}_j x) = \{q_0\} \). As there is no observable event before \( x \), \( \text{last}\_\text{modifier}(x) \) is either \( \perp_\pi(x) \) or a receive event. If \( \text{last}\_\text{modifier}(x) = \perp_\pi(x) \) then the property follows from S1. If \( \text{last}\_\text{modifier}(x) \) is a receive event, consider \( s \in E \) its associated send event. If \( \exists k : \Phi\text{pred}[k](\text{last}\_\text{modifier}(x)) \neq \{q_0\} \) then, due to statement S4, we have \( \Phi\text{pred}[k](s) \neq \{q_0\} \) and so \( \Phi\text{pred}[k]\perp_\pi(\text{modifier}(s)) \neq \{q_0\} \). Applying inductively this reasoning, there is a \( \perp_m \) such that \( \Phi\text{pred}[k]\perp_m \neq \{q_0\} \) which is impossible. So the proposition is true for \( x \in \mathcal{M}^0 \).

Induction case

Let \( r \) an integer, assume that the property is true for all rank \( r' < r \), we have to show:

\[ \forall x \in \mathcal{M}^r, \forall j \in 1..n : \Phi\text{pred}[j](\text{last}\_\text{modifier}(x)) = \Phi(\text{im} \text{pred}_j x) \]

**Lemma** \( \forall r' < r, \forall y \in \mathcal{M}^{r'}, \) we have \( \Phi\text{pred}[\pi(y)](y) = \Phi(y) \) and

\[ \forall j \neq \pi(y) : \Phi\text{pred}[j](y) = \{\}. \]

The Proof of this lemma follows from induction hypothesis and from statement S2.

Now, there are two cases to consider: \( \text{last}\_\text{modifier}(x) \) is an observable event or a receive event.
1. \texttt{last\_modifier}(x) is an observable event.

   It follows from definition of \texttt{last\_modifier}(x) that $\forall j \neq \pi(x) : \text{im\_pred}_{j} x = \{\}$ and $\text{im\_pred}_{\pi(x)} x = \{\text{last\_modifier}(x)\}$. Then the proposition follows directly from the previous lemma.

2. \texttt{last\_modifier}(x) is a receive event.

   Let $y = \text{max}(O_{\pi(y)} \cap \text{pred} x)$, in order to proof the proposition, we have to show the following property:

   \[
   \Phi\text{pred}[\pi(y)](\text{last\_modifier}(x)) = \{\} \iff \text{im\_pred}_{\pi(y)} x = \{\}
   \]

   \[
   \Phi\text{pred}[\pi(y)](\text{last\_modifier}(x)) = \Phi(y) \iff \text{im\_pred}_{\pi(y)} x = \{y\}
   \]

   which is equivalent to (by definition of \texttt{last\_modifier}(x)):

   \[
   \Phi\text{pred}[\pi(y)](\text{last\_modifier}(x)) = \{\} \iff \exists z \in O : y <_{E} z \land z <_{E} \text{last\_modifier}(x)
   \]

   \[
   \Phi\text{pred}[\pi(y)](\text{last\_modifier}(x)) = \Phi(y) \iff \nexists z \in O : y <_{E} z \land z <_{E} \text{last\_modifier}(x)
   \]

   Let $s \in E$ be the send event associated with \texttt{last\_modifier}(x). Let $t$ be the event \texttt{last\_modifier}(\texttt{last\_modifier}(x)) (see Figure 6).

   ![](image)

   Figure 6: Two cases to consider.

   The proposition is proved by considering the two cases:

   (a) \quad \nexists z \in O : y <_{E} z \land z <_{E} \text{last\_modifier}(x) \Rightarrow \Phi\text{pred}[\pi(y)](\text{last\_modifier}(x)) = \Phi(y);
(b) \( \exists z \in O : y \leq z \land z \leq y \) last\_modifier\((x) \Rightarrow \Phi_{pred}[\pi(y)](last\_modifier(x)) = \{\} \).

(case 2a)
\[ \not\exists z \in O : y < z \land z < y \) last\_modifier\((x) \Rightarrow \Phi_{pred}[\pi(y)](last\_modifier(x)) = \Phi(y) \]
By contradiction, assume \( \not\exists z \in O : y < z \land z < y \) last\_modifier\((x) \) (H1) and \( \Phi_{pred}[\pi(y)](last\_modifier(x)) \neq \Phi(y) \) (H2).
Considering statement S4 of the algorithm, there are three cases to consider:

i. \( V_{\pi(x)}[\pi(y)] < V_{\pi(x)}[\pi(y)] \)
then \( \Phi_{pred}[\pi(y)](s) \neq \Phi(y) \),
so we have \( \Phi_{pred}[\pi(y)](last\_modifier(s)) \neq \Phi(y) \).
If last\_modifier\((s) \in O \) then by H1 it follows that \( y = last\_modifier(s) \) and then \( \Phi_{pred}[\pi(y)](y) \neq \Phi(y) \) which contradicts the lemma.
If last\_modifier\((s) \) is a receive event then we have
\[ \not\exists z \in O : y < z \land z < y \) last\_modifier\((s) \]
and \( \Phi_{pred}[\pi(y)](last\_modifier(s)) \neq \Phi(y) \).
To prove there is a contradiction, it remains to show that
\[ \not\exists z \in O : y < z \land z < y \) last\_modifier\((s) \Rightarrow \Phi_{pred}[\pi(y)](last\_modifier(s)) \neq \Phi(y) \).

ii. \( V_{\pi(x)}[\pi(y)] > V_{\pi(x)}[\pi(y)] \)
then \( \Phi_{pred}[\pi(y)](t) \neq \Phi(y) \),
by applying the same reasoning as in case 2(i), we show that \( t \not\in O \),
so we have \( \not\exists z \in O : y < z \land z < t \) and \( \Phi_{pred}[\pi(y)](t) \neq \Phi(y) \).
It remains to show that
\[ \not\exists z \in O : y < z \land z < t \Rightarrow \Phi_{pred}[\pi(y)](t) \neq \Phi(y) \).

iii. \( V_{\pi(x)}[\pi(y)] = V_{\pi(x)}[\pi(y)] \)
then we have to consider two cases:
A. \( \Phi_{pred}[\pi(y)](t) = \{\} \),
as \( t \not\in O \) (by applying the same reasoning as in cases 2(ii) and 2(ii)),
It remains to show that \( \not\exists z \in O : y < z \land z < t \Rightarrow \Phi_{pred}[\pi(y)](t) \neq \Phi(y) \).
B. \( \Phi_{pred}[\pi(y)](t) \neq \{\} \),
then we have \( \Phi_{pred}[\pi(y)](t) \neq \Phi(y) \) and the same reasoning as in case 2(ii) applies.
So considering the finiteness hypothesis, applying recursively the reasoning will fall in the contradiction case: \( \Phi_{pred}[\pi(y)](y) \neq \Phi(y) \) which proves 2a.

(case 2b)
\[ \exists z \in O : y < z \land z < y \) last\_modifier\((x) \Rightarrow \Phi_{pred}[\pi(y)](last\_modifier(x)) = \{\} \]
By contradiction, assume \( \exists z \in O : y < z \land z < y \) last\_modifier\((x) \) (H3) and \( \Phi_{pred}[\pi(y)](last\_modifier(x)) \neq \{\} \) (H4).
Considering statement S4 of the algorithm, there are three cases to consider:
i. $V_{\pi(x)}[\pi(y)] < V_{\pi(s)}[\pi(y)]$
then $\Phi_{\text{pred}}[\pi(y)](s) \neq \{\}$,
so we have $\Phi_{\text{pred}}[\pi(y)](\text{last\_modifier}(s)) \neq \{\}$.
If $\text{last\_modifier}(s) \in O$ there are two cases: $\text{last\_modifier}(s) = y$ or $\text{last\_modifier}(s) \neq y$.
If $\text{last\_modifier}(s) \neq y$ then by statement S2 and by H4 it follow that
$\pi(s) = \pi(y)$,
and by H3, $y <_{\pi(y)} \text{last\_modifier}(s)$ which is impossible as
$y = \max(O_{\pi(y)} \cap \text{pred} x)$.
If $\text{last\_modifier}(s) = y$ then it contradicts H3.
If $\text{last\_modifier}(s)$ is a receive event, we have
$\exists z \in O : y <_E z \land z <_E \text{last\_modifier}(s)$,
as $\exists z \in O : y <_E z \land z <_E s$ implies (by H3)
$\exists z \in O : y <_E z \land z <_E \text{last\_modifier}(x)$
which implies $V_{\pi(x)}[\pi(y)] > V_{\pi(s)}[\pi(y)]$ contradicting the hypothesis on
timestamps of events.
To prove there is a contradiction, it remains to show that
$\exists z \in O : y <_E z \land z <_E \text{last\_modifier}(s) \Rightarrow$
$\Phi_{\text{pred}}[\pi(y)](\text{last\_modifier}(s)) = \{\}$.

ii. $V_{\pi(x)}[\pi(y)] > V_{\pi(s)}[\pi(y)]$
then $\Phi_{\text{pred}}[\pi(y)](t) \neq \{\}$.
If $t \in O$ there are two cases: $t = y$ or $t \neq y$.
If $t \neq y$ then by statement S2 and by H4 it follow that $\pi(y) = \pi(t)$,
then $y <_{\pi(y)} t$ which is impossible as $y = \max(O_{\pi(y)} \cap \text{pred} x)$.
If $t = y$ then it contradicts H3.
If $t$ is a receive event then we have $\exists z \in O : y <_E z \land z <_E t$.
It remains to show that $\exists z \in O : y <_E z \land z <_E t \Rightarrow$
$\Phi_{\text{pred}}[\pi(y)](t) = \{\}$.

iii. $V_{\pi(x)}[\pi(y)] = V_{\pi(s)}[\pi(y)]$
then $\Phi_{\text{pred}}[\pi(y)](t) \neq \{\}$ and $\Phi_{\text{pred}}[\pi(y)](s) \neq \{\}$
the same reasoning as in case 2(ii) applies.

So considering the finiteness hypothesis, applying recursively the reasoning
will fall in the contradiction cases: $y \neq \max(O_{\pi(y)} \cap \text{pred} x)$ or
$\exists z \in O : y <_E z \land z <_E \text{last\_modifier}(x)$ which implies 2b. \qed

5 Particular Cases

According to the position of observable events with respects to communication events,
several particular cases can be defined. In all these cases the analysis algorithm simplifies.
5.1 Non invisible process participation

In this case we assume there is always one observable event during any interval of a process beginning with a receive event and ending with a send event. This assumption, called non invisible participation, is described in Figure 7.

![Diagram](image)

Figure 7: Non invisible process participation.

This assumption has the following immediate consequence: when a process $P_i$ sends a message we have always:

$$\Phi_{pred_i}[k] := \{\}, \forall k \in 1..n, k \neq i$$

It follows that only the vector $V_i[1..n]$ and the set $\Phi_{pred_i}[i]$ have to be piggybacked by the message sent. Statement S4 can be simplified accordingly and becomes:

**S4:** When $P_i$ receives from $P_k$ a message piggybacking $V_k$ and $\Phi_{pred_k}[k]$:

\[
\forall j \neq k: \text{if } V_i[j] \leq V_k[j] \text{ then } \Phi_{pred_i}[j] := \{\}\text{ fi};
\]

\[
\text{if } V_i[k] < V_k[k] \text{ then } \Phi_{pred_i}[k] := \Phi_{pred_k}[k]\text{ fi};
\]

\[
\forall j \in 1..n: V_i[j] := max(V_i[j], V_k[j]);
\]

5.2 Non invisible communication

Here we assume all communication events are observed in the following way: there is always an observable event just before every send event or just after every receive event (by “just before” or “just after” we mean there is neither a send nor a receive event between). In that case each observable event has exactly one or two immediate predecessors: always one on the same process and in the case of two, another on the process which sent the last received message. In Figure 8, $x$ has two immediate predecessors $z$ and $y; x'$ as only $x$ as immediate predecessor ($(t, x')$ is not an edge on a “longest control flow”).

It follows from this simplifying assumption that the array $\Phi_{pred_i}[1..n]$ is no more necessary, a simple $\Phi_{pred_i}$ being sufficient to compute $\Phi(x)$ for each observable event $x$. The algorithm becomes for each process $P_i$:
Figure 8: Non invisible communication.

S1: initialization:
\[
\forall j \in 1..n : V_i[j] := 0; \\
\Phi_{pred_i} := \{q_0\};
\]

S2: When \( P_i \) produces an observable event \( x \):
\[
V_i[i] := V_i[i] + 1; \\
\Phi_{pred_i} := \{\delta(q, \lambda(x)), \forall q \in \Phi_{pred_i}\}; \\
\% \Phi(x) = \Phi_{pred_i} \%
\]

S3: When \( P_i \) sends a message to \( P_k \):
\[
V_i[1..n] \text{ and } \Phi_{pred_i} \text{ are added to the message;}
\]

S4: When \( P_i \) receives from \( P_k \) a message piggybacking \( V_k \) and \( \Phi_{pred_k} \):
\[
\text{case} \\
V_i[k] \geq V_k[k] \text{ then skip} \\
V_i[i] = V_k[i] \text{ then } \Phi_{pred_i} := \Phi_{pred_k} \\
\text{else } \Phi_{pred_i} := \Phi_{pred_i} \cup \Phi_{pred_k} \\
\text{end-case} \\
\forall j \in 1..n : V_i[j] := max(V_i[j], V_k[j]);
\]

5.3 Considering all paths

If we are interested not in the “longest control flows” as defined by \( C(x) \) for each observable event \( x \), but in control flows ending at \( x \) (cf. discussion in Section 3.1) vector clocks become useless (remember they are used to consider only immediate predecessors when a message is received). The analysis algorithm simplifies accordingly.
S1: initialization:
\[ \Phi_{\text{pred}_i} := \{q_0\} \]

S2: When \( P_i \) produces an observable event \( x \):
\[ \Phi_{\text{pred}_i} := \{\delta(q, \lambda(x)), \forall q \in \Phi_{\text{pred}_i}\} ; \]
\% \( \Phi(x) = \Phi_{\text{pred}_i} \%

S3: When \( P_i \) sends a message to \( P_k \):
add \( \Phi_{\text{pred}_i} \) to the message;

S4: When \( P_i \) receives from \( P_k \) a message piggybacking \( \Phi_{\text{pred}_k} \):
\[ \Phi_{\text{pred}_i} := \Phi_{\text{pred}_i} \cup \Phi_{\text{pred}_k} \]

6 Related works

The algorithm introduced in [4] to detect linked predicates and the algorithm which detects regular patterns in distributed computation introduced in [3] are two particular uses of the simplified algorithm described in Section 5.3 which considers all control flows ending at observable events.

The algorithm described in [1] that computes the immediate predecessors of an observable event \( x \) is a special case of the general analysis algorithm where all observable events have the same label \( l \) and the automaton has only one state \( q_0 \) with \( \delta(q_0, 1) = q_0 \). With such an automaton, when an observable event \( x \) is produced by \( P_i \) we have:
\[ (\exists y : \text{im}_{\text{pred}_j} x = \{y\}) \leftrightarrow \Phi_{\text{pred}_i}[j] \neq \{\} \]

7 Conclusion

A general algorithm working on the fly and without delay has been introduced to analyze distributed computations. It associates with each observable event \( x \) of the computation the set of the longest control flows (sequences of observable events) that terminates at this event. A labelling function allows the user to consider these sequences as words on some alphabet and the algorithm checks whether these words belong to some language (defined by a finite state automaton). It has been shown that according to the constraints on the position of observable events with respect to communication events, the analysis algorithm can be simplified.
References


