Gossiping in Directed Graphs

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Gossiping is a problem of information dissemination described in a group of individuals (or *nodes*) connected by a communication network, where every node in the network knows a unique item of information and needs to communicate it to everyone else. This communication pattern finds its main applications in the field of interconnection networks for parallel and distributed architecture. Here, we study a model where any two neighbouring nodes in the network communicate in only direction ; in that case, the networks are modeled by directed graphs and the model is said to be *half-duplex*. Moreover, at any time, a node can communicate with at most one of its neighbouring nodes (*1-port* model), and every communication takes one time unit, or *round*.

We first assume that the network is fully connected, ie there exists in our directed graph (or *digraph*) a directed link between every pair of nodes ; in other words, we deal with the complete symmetric digraph with n nodes, K_n^*. Here, we provide a survey about the minimum number of rounds $\vec{g}_n$ necessary to achieve gossiping in K_n^*, and show how to improve the computation of $\vec{g}_n$ in some cases. Indeed, surprisingly, $\vec{g}_n$ is not always precisely known, though we know that it is closely related to the k-th Fibonacci number $F(k)$. More precisely, $\vec{g}_n \in \{k, k+1, k+2\}$, where k is the smallest integer such that $F(k) \geq \lfloor \frac{n}{2} \rfloor$.

However, for some classes of values of n, $\vec{g}_n$ is precisely known. In that case, one can notice that not all the links are used in an optimal gossip scheme. Hence, it is interesting to minimize the number of links in the network, for cost reasons. A *gossip digraph* defines a digraph in which gossiping can be achieved in $\vec{g}_n$; a *Minimum Gossip Digraph*, or *MGD*, defines a gossip digraph with a minimum number of directed links. We present here several new techniques to construct sparse gossip digraphs on n nodes, which give us general lower and upper bounds on the number of directed links in *MGD*s. For some small values of n, our techniques directly produce *MGD*s. For other values of n, our techniques produce the sparsest known digraphs of this type.

Example : Suppose $n = 6$. In that case, $n = 2 \times F(\mathbf{4})$, and thus we know that $\vec{g}_6 = \mathbf{4}+1 = 5$. However, as shown in Figure 1, it is possible to achieve gossiping in 5 rounds without using all the directed links in K_6^*. More precisely, we can show that the digraph shown below is a *MGD* with 6 nodes.

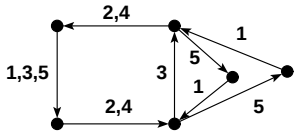

Figure 1: A *MGD* with 6 nodes

References

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