Chapter 1

Linguistic Meaning from the Language Acquisition Perspective

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ABSTRACT. From a plausible cognitive hypothesis explaining how little children develop complex meaning structures, we come to the conclusion that linguistic meanings are planned. The plans, we call them *discourse plans*, are tree-like structures composed of primitive situations and establishing referential links between the situations and the context. Linguistic meanings are second order Lambda-terms derived through conversion from homomorphic images of realizable discourse plans.

1.1 Introduction

There are two different approaches to meaning formalization: one *logical*, another *cognitive*. The logical approach, going back to Frege and developed and explicitly applied to natural language semantics by Montague, is characterized by establishing semantics from primitive semantic structures of *sentential type*. This approach is most consistently represented by type logical grammars (see collections (Gamut 1991; van Benthem and ter Meulen 1997)).

The cognitive approach treats linguistic meaning as a structure (e.g., a graph) *encoding the content* of a text in enough detail to represent it at the syntactic, morphological and phonological levels, independent of the propositional attitude. This is the way it appears in the theory "Meaning $\Leftrightarrow$ Text" (Mel'čuk 1997). This inspires the idea that this must be exactly the kind of meaning structures developed by the young children of the age under four years, when they learn their mother tongue. Indeed, in this age the children cannot rely upon an ontological knowledge, on a systematic reasoning of any kind, nor on any meta-knowledge, such as meaning subsumption, presumptions, etc. So they must develop meaning structures more or less in the same way as they develop the system of phonemes, i.e. discovering salient factors in the speech addressed to them, relating these factors to the speech context, and then unifying and generalizing their observations. Relying upon a psycho-linguistic literature on mother tongue acquisition by young children (cf. Gleitman and Liberman 1995; Hirsh-Pasek and Golinkoff 1996), we develop in (Dikovsky submitted) a hypothetical model of this inductive process and describe it

Proceedings of Formal Grammar 2003
G. Jäger, P. Monachesi, G. Penn & S. Wintner (editors).

there in much detail. In this paper, we state its main points and define the meaning elements developed at these points.

1.2 Roles, Types and Situations

The first link between the speech and its context is established by young children using several initial *cognitive roles* such as $\mathbf{ACT}$ (action), $\mathbf{AGT}$ (agent), $\mathbf{PAT}$ (patent), $\mathbf{LOC}$ (location), which mark certain positions in acoustic speech chunks. With time, the set of roles grows and their function changes (see below). We use in illustrations some of the following roles: $\mathbf{EVT}$ (event), $\mathbf{ST}$ (state), $\mathbf{TNS}$ (tense), $\mathbf{CAG}$ (counteragent), $\mathbf{OBJ}$ (object), $\mathbf{EXP}$ (experiencer), $\mathbf{ADR}$ (addresser), $\mathbf{ORG}$ (origin), $\mathbf{RCP}$ (recipient), $\mathbf{DST}$ (destination), $\mathbf{EFF}$ (effect), $\mathbf{GL}$ (goal), $\mathbf{CND}$ (condition), $\mathbf{CSE}$ (cause), $\mathbf{INS}$ (instrument), $\mathbf{RES}$ (result), $\mathbf{CLS}$ (class), $\mathbf{EL}$ (element), $\mathbf{PRO}$ (proprietor), $\mathbf{ATR}$ (attribute), $\mathbf{QUA}$(qualia), $\mathbf{DEF}$ (definiendum), $\mathbf{STR}$ (strength), $\mathbf{INT}$ (intensivity)[1].

Types. Linguistic meaning types originate from a rudimentary distributional analysis of acoustic speech chunks and positions marked by cognitive roles (see section 1.5 for more details).

Primitive types. *Primitive types* make a finite lattice $(\mathbf{P}, \prec)$ containing four initial pairwise incomparable elementary types:

− the type of *sententiator* $\mathbf{s}$ intuitively corresponding to “actions/processes/events”,

− the type of *nominators* $\mathbf{n}$ intuitively corresponding to “things” in the most general sense,

− the type of *qualifiers* $\mathbf{q}$, intuitively corresponding to the meanings qualifying the nominators,

− the type of *circumscriptors* $\mathbf{c}$, intuitively corresponding to the meanings, qualifying the qualifiers and the sententiators.

$\prec$ is an instance/generic partial order relation on primitive types. For example, the type of nominators $\mathbf{n}$ has the instances $\mathbf{n_a},\mathbf{n_{\tilde{a}}} \prec \mathbf{n}$ of *animated/inanimated nominators*, the type of sententiators $\mathbf{s}$ has the instance $\mathbf{s_{om}} \prec \mathbf{s}$ of *oriented movement sententiators* (e.g., $\mathbf{run_1}$, $\mathbf{mount}$), and also the instance $\mathbf{s_{bf}} \prec \mathbf{s}$ of *belief sententiators*, the type $\mathbf{q}$ of qualifiers has the instance $\mathbf{q_{cp}} \prec \mathbf{q}$ of *comparison qualifiers* (e.g., $\mathbf{better}$, $\mathbf{worst}$), and the instance $\mathbf{q_{qu}} \prec \mathbf{q}$ of *quantification qualifiers* (e.g., $\mathbf{all}$, $\mathbf{neither}$, $\mathbf{four}$), the type $\mathbf{c}$ of circumscriptors has the instance $\mathbf{c_{dg}} \prec \mathbf{c}$ of *degree circumscriptors* (e.g., $\mathbf{more}$, $\mathbf{especially}$).

Let $\bot = \wedge\mathbf{P}$ be the *least* and $\top = \vee\mathbf{P}$ be the *greatest* primitive types.

Complex types. The first observed complex types correspond to speech elements which may be present or not in a marked position (*option types*) and to sequences of speech elements of a primitive type, which occur in a marked position (*iterative types*).

Option types. $\mathbf{O} = \{\mathbf{u}^{(\mathbf{0})} \mid \mathbf{u} \in \mathbf{P} \setminus \{\bot, \top\}\}$ is the set of *option types*.

Iterative types. $\mathbf{I} = \{\mathbf{u}^{(\omega)} \mid \mathbf{u} \in \mathbf{P} \setminus \{\bot, \top\}\}$ is the set of *iterative types*.

[1] Of course, this list is far from being complete and minimal.

Basic types. $\mathbf{B} = \mathbf{P} \cup \mathbf{O} \cup \mathbf{I}$ is the set of *basic types*. The type instance order $\prec$ is naturally extended to $\mathbf{B} \setminus \{\bot, \top\}$: $\mathbf{u}^{(\mathbf{0})} \prec \mathbf{v}^{(\mathbf{0})}$ and $\mathbf{u}^{(\omega)} \prec \mathbf{v}^{(\omega)}$ for all $\mathbf{u} \prec \mathbf{v}$ in $\mathbf{B} \setminus \{\bot, \top\}$.

Being combined with nominators, the qualifiers give new nominators. Being combined with qualifiers, the circumscriptors form new qualifiers, and being combined with sententiators, they form new sententiators. From this follows the particularity, that all words have functional meaning types with an iterative subtype. For instance, common nouns have type $(\mathbf{q}^{(\omega)} \rightarrow \mathbf{n})$. They are interpreted by functions from lists of qualifier type meanings to nominator type meanings. Adjectives have type $(\mathbf{c}^{(\omega)} \rightarrow \mathbf{q})$. They are interpreted by functions from lists of circumscriptor type meanings to qualifier type meanings. Adverbs have type $(\mathbf{c}^{(\omega)} \rightarrow \mathbf{c})$. Their meanings are functions from lists of circumscriptor type meanings to circumscriptor type meanings. Non-functional meanings are of only two kinds: $\emptyset^{\mathbf{u}^{(\mathbf{0})}}$ (*empty meaning* of option type $\mathbf{u}^{(\mathbf{0})}$) and $\mathbf{nil}^{\mathbf{u}^{(\omega)}}$ (*empty list* of type $\mathbf{u}^{(\omega)}$). This is why, we distinguish between iterative type and non-iterative type function arguments, the latter being named *actants*. Children learn verb actants before the age of two. First, they are identified as arguments of causal (transitive) and non-causal (intransitive) verb meanings. Later, it turns out that words with meanings of other value types also have actants. For instance, the nominator, which is the meaning of the noun '$PART$', has two nominator type actors (*what*) and (*of-what*). So it has the functional type $(\mathbf{q}^{(\omega)}\mathbf{nn} \rightarrow \mathbf{n})$.

Notation. The expression $(\mathbf{u}_1\mathbf{u}_2...\mathbf{u}_k \rightarrow \mathbf{v})$ denotes the type $(\mathbf{u}_1 \rightarrow (\mathbf{u}_2 \rightarrow ... (\mathbf{u}_k \rightarrow \mathbf{v})...))$.

Meanings with actants are the first primitive meaning structures. We call them *situations*.

Situation types. There are four families of *situation types*:
$\mathbf{T}^{\mathbf{s}} = \{(\mathbf{c}^{(\omega)}\mathbf{u}_1...\mathbf{u}_r \rightarrow \mathbf{s}) \mid \mathbf{u}_i \in \mathbf{P} \cup \mathbf{O}, r \geq 0\}$,
$\mathbf{T}^{\mathbf{n}} = \{(\mathbf{q}^{(\omega)}\mathbf{u}_1...\mathbf{u}_r \rightarrow \mathbf{n}) \mid \mathbf{u}_i \in \mathbf{P} \cup \mathbf{O}, r \geq 0\}$,
$\mathbf{T}^{\mathbf{q}} = \{(\mathbf{c}^{(\omega)}\mathbf{u}_1...\mathbf{u}_r \rightarrow \mathbf{q}) \mid \mathbf{u}_i \in \mathbf{P} \cup \mathbf{O}, r \geq 0\}$,
$\mathbf{T}^{\mathbf{c}} = \{(\mathbf{c}^{(\omega)}\mathbf{u}_1...\mathbf{u}_r \rightarrow \mathbf{c}) \mid \mathbf{u}_i \in \mathbf{P} \cup \mathbf{O}, r \geq 0\}$.
$\mathbf{T} = \mathbf{B} \cup \mathbf{T}^{\mathbf{s}} \cup \mathbf{T}^{\mathbf{n}} \cup \mathbf{T}^{\mathbf{q}} \cup \mathbf{T}^{\mathbf{c}}$ is the set of *types*.

Situations are defined in the dictionary by *situation profiles* consisting of the situation's key, of its type, of the basic lexeme and of roles of its actants. For instance, the situation, which is the meaning of the verb lexeme 'PUT_1', has profile

$$\mathbf{sit}(\mathbf{put_1}^{(\mathbf{c}^{(\omega)}\mathbf{n_a}\mathbf{nc} \rightarrow \mathbf{s})}, `PUT_1\text{'}(\mathbf{AGT}(1), \mathbf{PAT}(2), \mathbf{LOC}(3)))$$

identified by the key $\mathbf{put_1}$, stating that this situation has three obligatory actants: the first actant of type $\mathbf{n_a}$ has the role $\mathbf{AGT}$, the second of type $\mathbf{n}$ has the role $\mathbf{PAT}$, the third of type $\mathbf{c}$ has the role $\mathbf{LOC}$. Besides them, it has the standard iterative circumscriptor-type argument. This situation has value type $\mathbf{s}$.

Three-four months after their second birthday, children try to compose complex meanings of situations. This corresponds to Lambda abstraction of situation arguments. For instance, the profile of $\mathbf{put_1}$ induces the Lambda term[2]:

$$\lambda Y^{\mathbf{c}^{(\omega)}} X_1^{\mathbf{n_a}} X_2^{\mathbf{n}} X_3^{\mathbf{c}}.\ \mathbf{put}_1(Y, X_1, X_2, X_3).$$

Besides this, they start using pronouns co-referential with full noun phrases (so they use context).

1.3 Discourse Plans

Composing abstract situations, children meet with the following three problems. First is that the type of the actant of the host situation which should be developed using a new situation may conflict with the value type of the latter. For instance, in order to use a sententiator-type meaning as a qualifier-type argument of a nominator (cf. relative clauses modifying names), the type transformation $(\mathbf{s} \Rightarrow \mathbf{q})$ should be effected somehow. The second problem is more complex and is often due to the intended communicative organization of the complex meaning. It arises when the new situation should be a semantic derivative of some "clue" situation, as it is the case of the voices of verbs. In order to convert the original situation into its derivative, semantic analogues are needed of the voice diatheses (Mel'čuk and Holodovich 1970). The third problem has been the subject of long debate in the literature (see (van Benthem and ter Meulen 1997)). This is the problem of using references to a meaning unit outside of the scope of its definition. For instance, in the example of Geach:

If a farmer has a donkey, he beats it

the anaphorical pronouns corresponding to the actors of situation $beat$ reference the corresponding actors of situation $have$, whereas their scopes are disjoint. More complex are implicit tense relations, as in the following Kamp's sentence:

A child was born that will become ruler of the world.

To surmount these obstacles, the child acquires planning of complex meanings. The plans, we call them *discourse plans (DPs)*, are defined by the grammar in Fig. 1.1.

A DP can be seen as a sequence of *plan points* and also as a hierarchy of *sub-plans*. These two orders induce the corresponding infix order on sub-plans. Each plan point introduces into DP a new primitive, or a new aggregate, or a situation's derivative defined by a situation converter. They all can be referenced. Long term references are kept in global context Γ, which is never updated. Short-term references added at some points of DP to local context ρ can be used and deleted from ρ later in DP. The scope and visibility of local references in the DP is defined in terms of the infix order on sub-plans.

Primitives are of four kinds:

(p_1) Empty lists of qualifiers and circumscriptors $\mathbf{nil}^{\mathbf{q}^{(\omega)}}$, $\mathbf{nil}^{\mathbf{c}^{(\omega)}}$ and null values of option types: $\emptyset^{\mathbf{u}^{(\mathbf{0})}}$ (both omitted in examples).

(p_2) Actant-less lexical units, for instance, common nouns such as 'CAT'$^{(\mathbf{q}^{(\omega)} \rightarrow \mathbf{n_a})}$.

[2] We abbreviate $\lambda X_1.\ (...(\lambda X_n.\ X_n.\ t)...)$ by $\lambda X_1...X_n.\ t$.

$\mathbf{DP}$	::=	$\mathbf{SP}^+$	(discourse plan)
$\mathbf{SP}$	::=	$\mathbf{Prim} \mid$	(sub-plan)
		$\mathbf{A_{op}}\{\mathbf{AComps}\} \parallel$	(aggregate)
		$\mathbf{Key\ Conv\ Mode}\ \{\mathbf{DOs}\}$	(situation)
$\mathbf{DOs}$	::=	$\mathbf{DO} \parallel \mathbf{DO}, \mathbf{DOs}$	(development operators)
$\mathbf{DO}$	::=	$\mathbf{Arg} \Leftarrow \mathbf{SP}$	(development operator)
$\mathbf{AComps}$	::=	$\mathbf{SP}, \mathbf{SP} \mid \mathbf{SP}, \mathbf{AComps}$	(aggregate components)
$\mathbf{Prim}$			(primitive)
$\mathbf{Conv}$			(converter)
$\mathbf{Mode}$			(intonation marker)
$\mathbf{Key}$			(situation identifier)
$\mathbf{Arg}$			(situation argument)
$\mathbf{op}$			(aggregation operator)

Figure 1.1: DP syntax

(p_3) Embedded relations, e.g. tense relations.
(p_4) Context access operators:
$(\Uparrow x)^{\mathbf{u}}$ co-referential with an element of type $\mathbf{u}$ in the local context;
$(\Downarrow_x^{\mathbf{u}}\ LEX)^{\mathbf{u}}$ creating and adding to the local context a new reference x of type $\mathbf{u}$ to the lexical unit LEX;
$(\Uparrow^{\mathbf{g}}\ N)^{\mathbf{u}}$ accessing the global context element of type $\mathbf{u}$ identified by N;
$(\Uparrow_x^{\mathbf{g}}\ N)^{\mathbf{u}}$ creating and adding to the local context a reference x to the global context element identified by N (x has the type of N).

Converters are fundamental means of planning. A converter **conv** used in a development operator $\mathbf{Arg} \Leftarrow \mathbf{sub\!-\!plan}$, in which

$$\mathbf{sub\!-\!plan} = \mathbf{key\ conv\ mode}\ \{\mathbf{sub\!-\!plans}\},$$

determines a semantic derivative $\mathbf{der}(\mathbf{key}, \mathbf{conv})$ of situation $\mathbf{key}$ to be composed in the argument Arg of the host situation. So it is a second order operator applied to situation's meaning. For any two types $\mathbf{u}, \mathbf{v} \in \mathbf{T}$, $(\mathbf{u} \Rightarrow \mathbf{v})$ is a *converter type*. Below, we outline three kinds of converters: *TR-Converters, abstractors* and *direct diatheses*.

TR-converters relate to situations and to primitives their inherent attributes determined by roles and constrained by types. Intuitively, the role $\mathbf{R}$ of a $\mathbf{uR}$-converter applied to a DP element E determines an inherent attribute of the meaning of E. The type $\mathbf{u}$ of this converter is one of the possible types of values of this attribute. For instance, each situation with $\mathbf{s}$-type value has a semantic tense attribute $\mathbf{TNS}$, and each nominator expressing a physical body has a location attribute $\mathbf{LOC}$. Respectively, the tense TR-converter has the form $(\ )^{\mathbf{u}\ \mathbf{TNS}}$, where $\mathbf{u}$ is a tense circumscriptor type (e.g. $\mathbf{t_{ntr}} \prec \mathbf{t}$: neutral (gnomical) tense, $\mathbf{t_{pnt}} \prec \mathbf{t}$: pointwise tense, $\mathbf{t_{int}} \prec \mathbf{t}$: interval tense), and the location TR-converter has the form $(\ )^{\mathbf{v}\ \mathbf{LOC}}$, where $\mathbf{v}$ is a location circumscriptor type. For example, let ρ contain a reference s_2 to the situation:

$$\mathbf{sit}(\mathbf{be_3}^{(\mathbf{c}^{(\omega)}\mathbf{nn}\to\mathbf{s})}, \text{‘}BE_3\text{’}(\mathbf{EL}(1), \mathbf{CLS}(2))).$$

Then in the DP fragment:

$[t]$ $\quad \mathbf{t_{prg}} \Leftarrow \Downarrow_{t_1}^{\mathbf{t_{prg}}} (\Uparrow s_2)^{\mathbf{t_{prg}}}\ \mathbf{TNS}$,

$[t+1]$ $\quad \mathbf{t} \Leftarrow ((\Uparrow^{\mathbf{g}}\ \mathbf{now}) \in t_1)^{\mathbf{t}}$, % $(\Uparrow^{\mathbf{g}}\ \mathbf{now})$: the moment of speech

TR-converter $(\Uparrow s_2)^{\mathbf{t_{prg}}}\ \mathbf{TNS}$ defines the tense attribute of situation $\mathbf{be_3}$ with value type $\mathbf{t_{prg}}$. At point $[t]$, the operator $\Downarrow_{t_1}^{\mathbf{t_{prg}}} (\Uparrow s_2)^{\mathbf{t_{prg}}}\ \mathbf{TNS}$, adds to ρ the reference t_1 to this attribute. Besides this, at this point, $(\Uparrow s_2)^{\mathbf{t_{prg}}}\ \mathbf{TNS}$ becomes a circumscriptor of $\mathbf{be_3}$ of type $\mathbf{t_{prg}}$. At the next point, the reference t_1 is used to introduce a new circumscriptor of $\mathbf{be_3}$, which states the semantic present tense.

Abstractor ($\mathbf{abs}^x$) has the approximate meaning "*such x that*". Abstractors are used at the plan points of the form:

$$\mathbf{q} \Leftarrow \mathbf{key_1}^{(\bar{\mathbf{u}}\to\mathbf{v})}\ \mathbf{abs}^x\ \{\mathbf{sub\!-\!plans}\},$$

where the situation $\mathbf{key}_1$ develops a qualifier type argument of its host situation $\mathbf{key}_2$. So $\mathbf{abs}^x$ describes type conversions $(\mathbf{v}\ \Rightarrow\ \mathbf{q})$.

Direct diathesis ($\mathbf{dth}$) determines how the type and the argument structure of the introduced situation should be adapted to the developed argument type and role and to the new positions and roles of its actants. For instance, applied to situation:
$\mathbf{sit}(\mathbf{bear_2}^{(\mathbf{c}^{(\omega)}\mathbf{n_a}\mathbf{n_a}\to\mathbf{s})}, \text{‘}GIVE\_BIRTH\text{’}(\mathbf{AGT}(1)^{\mathbf{n_a}}, \mathbf{PAT}(2)^{\mathbf{n_a}}))$,
the diathesis $\mathbf{dth_1}(\mathbf{bear_2}) = (\mathbf{PAT}(2)^{\mathbf{n_a}}, \mathbf{AGT}(1)^{\mathbf{n_a^{(0)}}})^{\mathbf{s}}\ \mathbf{EVT}$
determines the semantic derivative: $\mathbf{der}(\mathbf{bear_2}, \mathbf{dth_1}) =$
$\mathbf{sit}(\mathbf{be\_born}^{(\mathbf{c}^{(\omega)}\mathbf{n_a}\mathbf{n_a^{(0)}}\to\mathbf{s})}, \text{‘}GIVE\_BIRTH\text{’}(\mathbf{PAT}(2), \mathbf{AGT}(1)))$.
Applied to situation:
$\mathbf{sit}(\mathbf{rule_1}^{(\mathbf{c}^{(\omega)}\mathbf{n_a}\mathbf{n}\to\mathbf{s})}, \text{‘}RULE_1\text{’}(\mathbf{AGT}(1)^{\mathbf{n_a}}, \mathbf{PAT}(2)^{\mathbf{n}}))$,
the diathesis $\mathbf{dth_2}(\mathbf{rule_1}) = (\mathbf{PAT}(2)^{\mathbf{n}^{(0)}})^{\mathbf{n_a}}\ \mathbf{AGT}$ determines:
$\mathbf{der}(\mathbf{dth_2}, \mathbf{rule_1}) = \mathbf{sit}(\mathbf{ruler\_of}^{(\mathbf{q}^{(\omega)}\mathbf{n}\to\mathbf{n_a})}, \text{‘}RULE_1\text{’}(\mathbf{PAT}(2)))$.
In general, a direct diathesis

$$\mathbf{dth}(\mathbf{key}) = (\mathbf{R_1'}(j_1)^{\mathbf{u}_1'}, ..., \mathbf{R_k'}(j_k)^{\mathbf{u}_k'})^{\mathbf{v}'}\ \mathbf{R}$$

applied to a situation $\mathbf{key}$ with profile

$$\mathbf{sit}(\mathbf{key}^{(\mathbf{u}^{(\omega)}\mathbf{u}_1...\mathbf{u}_n\to\mathbf{v})}, \text{‘}LEXEME\text{’}(\mathbf{R_1}(1), ..., \mathbf{R_n}(n)))$$

determines a derivative with profile

$$\mathbf{sit}(\mathbf{key}'^{(\mathbf{u}'^{(\omega)}\mathbf{u}_1'...\mathbf{u}_k'\to\mathbf{v}')}, \text{‘}LEXEME\text{’}(\mathbf{R_1'}(j_1), ..., \mathbf{R_k'}(j_k))).$$

This derivative is introduced in sub-plan of type $\mathbf{v}'$ for role $\mathbf{R}$ by development operators:

$[t]$ $\quad \mathbf{Arg} \Leftarrow \mathbf{key}^{(\mathbf{u}^{(\omega)}\mathbf{u}_1...\mathbf{u}_n\to\mathbf{v})}\ \mathbf{dth}\ \{$

$[t+p]$ $\quad\quad (j_1)^{\mathbf{u}_1'} \Leftarrow \{sub\!-\!plan_1\}^{\mathbf{u}_1'\mathbf{R}_1'}$,

...

$[t+r]$ $(j_k)^{\mathbf{u}'_k} \Leftarrow \{sub{-}plan_k\}^{\mathbf{u}'_\mathbf{k}\mathbf{R}'_\mathbf{k}}$

...}

where each $\{sub{-}plan_i\}^{\mathbf{u}'_\mathbf{i}\ \mathbf{R}'_\mathbf{i}}$ is a sub-plan of type $\mathbf{u}'_\mathbf{i}$ for role $\mathbf{R}'_\mathbf{i}$.

Co-reference planning. ρ is a bounded resource memory. Adding to ρ a local reference may cause deletion of some other references. Reference deletion may occur not only for the reason of bounded memory size, but also because this memory cannot keep at the same plan point two "similar" "independent" references. We propose the following notion of reference similarity. When a local reference x is created in expression $\Downarrow_x^{\mathbf{u}}$ $\mathbf{key\ Conv}$ $\{Args\}$, it is assigned the *format* $fm(x) = (\mathbf{u}, \mathbf{R})$, where $\mathbf{u}$ is the type of x and $\mathbf{R}$ is the role assigned to the sub-plan $\mathbf{key\ Conv}$ $\{Args\}$ by the development operator in which it is used. If the role is not assigned, this will be the most general *undefined role* $\Re$. A reference x is *visible* at point $[i]$ of DP if $x \in \rho$ at this point. Visibility is determined by the following rules ($\leq_c$ is the infix order of sub-plans):

$(\mathbf{v_1})$ There is a constant κ limiting the number of visible references in ρ Adding to ρ $(\kappa+1)$-st reference causes **deletion** from ρ of the earliest[3] reference.

$(\mathbf{v_2})$ Expressions

$$\Downarrow_x^{\mathbf{u}} \mathbf{key\ Conv}\ \{\mathbf{Args}\} \text{ and } (\Uparrow_x^{\mathbf{g}} \mathbf{key}^{\mathbf{u}})$$

add the reference x [4] to ρ and at the same time **remove** from ρ any other similar local reference y, on which x does not c-depend: $y \not\leq_c x$.

$(\mathbf{v_3})$ Local context access/update operators are executed in the order of the plan points where they are used.

$(\mathbf{v_4})$ In operators $\mathbf{q} \Leftarrow SP$, $\mathbf{c} \Leftarrow SP$ developing in DP π a qualifier/circumscriptor argument of a situation (of a primitive) S, any reference $\Uparrow_x$ used in SP is c-dependent only on those elements in π on which S c-depends (in particular, on S itself).

$(\mathbf{v_5})$ A reference to a situation argument is removed together with the references to all this argument's qualifiers/circumscriptors.

Communicative structure. Diatheses have effect on communicative structure. Namely, the actant promoted to the first actant position of derivative S becomes its *theme*. The *rheme* of S will be the set of all other *locally referenced arguments* of S. Intuitively, the rheme consists of those arguments related to the theme through S, which can be passed as parameters to situations introduced later in the DP. A DP has a unique point marked by the focus $\odot$. The DP element introduced at the focalized point must be locally referenced.

Let us plan the meaning of the Kamp's sentence using the two diatheses above.

Discourse plan π:

[1] $\Downarrow_{S_1}^{\mathbf{S}}$ $\mathbf{bear_2}^{(\mathbf{c}^{(\omega)}\mathbf{n_a n_a} \to \mathbf{S})}$ $\mathbf{dth_1}$ {

[3] I.e. with the least DP point number.

[4] Never used at the preceding plan points.

[2] $\mathbf{t_{pnt}} \Leftarrow \Downarrow_{t_1}^{\mathbf{t_{pnt}}} (\Uparrow s_1)^{\mathbf{t_{pnt}}}\ \mathbf{TNS},$

[3] $\mathbf{t} \Leftarrow (\Uparrow t_1 \triangleleft (\Uparrow^{\mathbf{g}}\ \mathbf{now}))^{\mathbf{t}},$ % before now

[4]$\odot$ $(2)^{\mathbf{n_a}} \Leftarrow \Downarrow_{n_1}^{\mathbf{n_a}}$ '$CHILD$'$^{(\mathbf{q}^{(\omega)} \to \mathbf{n_a})}\{$

[5] $\mathbf{q} \Leftarrow \Downarrow_{q_1}^{\mathbf{q}} \Downarrow_{s_2}^{\mathbf{s}}\ \mathbf{be_3}^{(\mathbf{c}^{(\omega)}\mathbf{nn} \to \mathbf{s})}\ \mathbf{abs}^{n_1}\ \{$

[6] $\mathbf{t_{pnt}} \Leftarrow \Downarrow_{t_2} (\Uparrow s_2)^{\mathbf{t_{pnt}}}\ \mathbf{TNS},$

[7] $\mathbf{t} \Leftarrow (\Uparrow t_1 \triangleleft \Uparrow t_2)^{\mathbf{t}},$ % t_2 after t_1

[8] $(1)^{\mathbf{n_a}} \Leftarrow (\Uparrow n_1)^{\mathbf{n_a}},$

[9] $(2)^{\mathbf{n}} \Leftarrow \Downarrow_{n_2}^{\mathbf{n}} \mathbf{rule}^{(\mathbf{c}^{(\omega)}\mathbf{n_a}\mathbf{n} \to \mathbf{s})}\ \mathbf{dth_2}\ \{$

[10] $(2)^{\mathbf{n}} \Leftarrow$ '$WORLD_1$'$^{(\mathbf{q}^{(\omega)} \to \mathbf{n})}$

$\}\ \}\ \}\ \}$

This DP is consistent in the following sense:

Definition 1 *A DP is* s(cope)-consistent *if all local references used in this plan are visible at the points, where they are used*[5]*.*

Moreover, π is *realizable*, in the sense that all the planned diatheses have the corresponding semantic derivatives in English (see above).

1.4 Meanings

Meanings are Lambda terms derived from realizable DPs by means of the following simple translation τ defined by induction on sub-plans S.

Definition 2 $(\mathbf{m_1})$ $\tau(S) = S$ *if* S *is a primitive sub-plan.*
$(\mathbf{m_2})$ $\tau(\odot S) = \odot\tau(S)$.
$(\mathbf{m_3})$ $\tau((S_1, S_2)) = (\tau(S_1), \tau(S_2))$.
$(\mathbf{m_4})$ $\tau(\mathbf{A_{op}}\{Comps\}) = \mathbf{A_{op}}\{\tau(Comps)\}$.
$(\mathbf{m_5})$ *Let* S *be a sub-plan of the form:*

$\mathbf{key\ Conv\ Mode}\ \{$
$(j_1)^{\mathbf{v}_1} \Leftarrow S_1,$
...
$(j_k)^{\mathbf{v}_k} \Leftarrow S_k,$
...
$\mathbf{u}_1 \Leftarrow S_{k+1,1},$
...
$\mathbf{u}_l \Leftarrow S_{k+1,l},$
...$\}$

and let $\mathbf{der}(\mathbf{key}, \mathbf{Conv}) = \mathbf{key_{der}}^{(\mathbf{u}^{(\omega)}\mathbf{v}_1 \ldots \mathbf{v}_n \to \mathbf{v})}$, $\mathbf{u}_m \preceq \mathbf{u}\ (1 \le m \le l)$, *be the semantic derivative of* $\mathbf{key}$ *determined by* $\mathbf{Conv}$. *Then, using the abstract*

[5]There is a real-time algorithm checking this property.

situation

$$\alpha(\mathbf{key}, \mathbf{Conv}) = \lambda X_0^{\mathbf{u}^{(\omega)}} X_1^{\mathbf{v}_1} ... X_k^{\mathbf{v}_k}.\ \mathbf{key_{der}}(X_0, X_1, ..., X_k),$$

the translation of S is defined as $\tau(S) =$

$\mathbf{sit}\ \mathbf{Mode}\,(\alpha(\mathbf{key}, \mathbf{Conv})[\tau(S_{k+1,1}), ..., \tau(S_{k+1,l})], \mathbf{act}_1(\tau(S_1)), ..., \mathbf{act}_k(\tau(S_k)))$.

The meaning derived from a realizable DP π *is the normal form term M such that $\tau(\pi) \twoheadrightarrow M$ (see below the point* **Reducibility***).*

Proposition 1 *1. If π is a realizable DP, then there exists a Lambda term $\tau(\pi)$. Moreover, for each sub-plan S of π of a primitive type $\mathbf{u}$, $\tau(S)$ also has type $\mathbf{u}$.*
2. If π is s-consistent, then the meaning derivable from π is also s-consistent.

The meaning of the sentence of Kamp derivable from π has the form:

(1) $\Downarrow_{s_1}^{\mathbf{s}}\ \mathbf{sit}(\mathbf{be\_born}^{(\mathbf{c}^{(\omega)}\mathbf{n_a}\mathbf{n_a^{(0)}} \to \mathbf{s})}$
(2) $[(\Downarrow_{t_1}^{\mathbf{t_{pnt}}} \Uparrow s_1)^{\mathbf{t_{pnt}}},$
(3) $(\Uparrow t_1 \triangleleft (\Uparrow^{\mathbf{g}}\ \mathbf{now}))^{\mathbf{t}}]^{\mathbf{c}^{(\omega)}},$
(4)$\odot$ $\mathbf{act}_2(\Downarrow_{n_1}^{\mathbf{n_a}}$ '$CHILD$'$^{(\mathbf{q}^{(\omega)} \to \mathbf{n_a})}($
(5) $[\Downarrow_{q_1}^{\mathbf{q}} \Downarrow_{s_2}^{\mathbf{s}}\ \mathbf{sit}(\mathbf{der}(\mathbf{be_3}, \mathbf{abs}^{n_1})$
(6) $[(\Downarrow_{t_2}^{\mathbf{t_{pnt}}} \Uparrow s_2)^{\mathbf{t_{pnt}}},$
(7) $(\Uparrow t_1 \triangleleft \Uparrow t_2)^{\mathbf{t}}]^{\mathbf{c}^{(\omega)}},$
(8) $\mathbf{act}_1((\Uparrow n_1)^{\mathbf{n_a}}),$
(9) $\mathbf{act}_2(\Downarrow_{n_2}^{\mathbf{n}}\ \mathbf{sit}(\mathbf{ruler\_of}^{(\mathbf{q}^{(\omega)}\mathbf{n} \to \mathbf{n_a})}$
(10) $\mathbf{act}_2($'$WORLD_1$'$^{(\mathbf{q}^{(\omega)} \to \mathbf{n})})$
$))\ \ )]^{\mathbf{q}^{(\omega)}}\ \ ))\ \ )$

We can now define the language of meanings $\mathcal{MT}^{\mathbf{lt}}$ (for space reasons, we don't include aggregates, modes, roles and foci).

Variables. We suppose that there are disjoint countable sets $\mathbf{V}^{\mathbf{u}}$ of *object variables* of type $\mathbf{u}$, $\mathbf{u} \in (\mathbf{B} \setminus \{\top\})$, and $\mathbf{R}^{\mathbf{u}}$ of *context variables* of type $\mathbf{u}$, for $\mathbf{u} \in \mathbf{T}$. We use upper-case latin indexed letters for the former and lower-case latin indexed letters for the latter.

Constants. For each type $\mathbf{u} \in \mathbf{T}$, $\mathbf{C}^{\mathbf{u}}$ is a countable set of *constants* of type $\mathbf{u}$. For each primitive type $\mathbf{u} \in (\mathbf{P} \setminus \{\bot, \top\})$, $\mathbf{C}^{\mathbf{u^{(0)}}} = \{\emptyset^{\mathbf{u^{(0)}}}\}$ ($\emptyset^{\mathbf{u^{(0)}}}$ is the *empty element* of type $\mathbf{u^{(0)}}$) and $\mathbf{C}^{\mathbf{u}^{(\omega)}} = \{\mathbf{nil}^{\mathbf{u}^{(\omega)}}\}$ ($\mathbf{nil}^{\mathbf{u}^{(\omega)}}$ is the *empty list* of type $\mathbf{u}^{(\omega)}$).

Converters. For each converter type $\mathbf{v} = (\mathbf{u}_1 \Rightarrow \mathbf{u}_2), \mathbf{u}_1, \mathbf{u}_2 \in \mathbf{T}$, $\mathbf{\Omega}^{\mathbf{v}}$ is a countable set of *converter constants* of converter type $\mathbf{v}$.

Context operators. $\mathbf{\Phi}$ is a countable set of *context operator names*.

Terms. The set $\mathcal{MT}^{\mathbf{lt}} =_{df} \bigcup_{\mathbf{u} \in \mathbf{T}} \mathcal{T}^{\mathbf{u}}$ of typed terms is the least set verifying the following conditions:

$(\mathbf{t_0})$ If $t = X \in \mathbf{V}^{\mathbf{u}}$, then $t \in \mathcal{T}^{\mathbf{u}}$ and $FV(t) = \{X\}$.
$(\mathbf{t_1})$ If $\mathbf{k} \in \mathbf{C}^{\mathbf{u}}$, then $\mathbf{k} \in \mathcal{T}^{\mathbf{u}}$ and $FV(\mathbf{k}) = \emptyset$.
$(\mathbf{t_2})$ If $\phi \in \mathbf{\Phi}$ and $x \in \mathbf{R}^{\mathbf{u}}$, then $t = \phi_x \in \mathcal{T}^{\mathbf{u}}$ and $FV(t) = \emptyset$.

$(\mathbf{t_3})$ If $\mathbf{u}^{(0)} \in \mathbf{O}$, then $\mathcal{T}^{\mathbf{u}} \cup \{\emptyset^{\mathbf{u}^{(\mathbf{0})}}\} \subseteq \mathcal{T}^{\mathbf{u}^{(\mathbf{0})}}$ and $FV(\emptyset^{\mathbf{u}^{(\mathbf{0})}}) = \emptyset$.
$(\mathbf{t_4})$ If $\mathbf{u} \in \mathbf{P}$ and $\mathbf{u}_1, ..., \mathbf{u}_k \preceq \mathbf{u}$, $k > 0$, then:
(i) $\mathbf{nil}^{\mathbf{u}^{(\omega)}} \in \mathcal{T}^{\mathbf{u}^{(\omega)}}$, $FV(\mathbf{nil}^{\mathbf{u}^{(\omega)}}) = \emptyset$, and
(ii) $t = [t_1, ..., t_k] \in \mathcal{T}^{\mathbf{u}^{(\omega)}}$ and $FV(t) = \bigcup_{i=1}^{k} FV(t_i)$ for any $t_1 \in \mathcal{T}^{\mathbf{u}_1}, ..., t_k \in \mathcal{T}^{\mathbf{u}_k}$, such that $FV(t_i) \cap FV(t_j) = \emptyset$, $1 \leq i \neq j \leq k$.
$(\mathbf{t_5})$ If $\gamma \in \mathbf{\Omega}^{(\mathbf{u} \Rightarrow \mathbf{v})}$ and $t \in \mathcal{T}^{\mathbf{u}}$, then $t_1 = \gamma\{t\} \in \mathcal{T}^{\mathbf{v}}$ and $FV(t_1) = FV(t)$.
$(\mathbf{t_6})$ If $t_0 \in \mathcal{T}^{(\mathbf{u} \rightarrow \mathbf{v})}$, $t_1 \in \mathcal{T}^{\mathbf{u}_1}$ for some $\mathbf{u}_1 \preceq \mathbf{u}$, and $FV(t_0) \cap FV(t_1) = \emptyset$, then $t = (t_0\ t_1) \in \mathcal{T}^{\mathbf{v}}$ and $FV(t) = FV(t_0) \cup FV(t_1)$.
$(\mathbf{t_7})$ If $t_0 \in \mathcal{T}^{\mathbf{v}}$, $(\mathbf{u} \rightarrow \mathbf{v}) \in \mathbf{T} \setminus \mathbf{B}$, and $X \in \mathbf{V}^{\mathbf{u}} \cap FV(t_0)$, then
$$t = \lambda X.\, t_0 \in \mathcal{T}^{(\mathbf{u} \rightarrow \mathbf{v})} \text{ and } FV(t) = FV(t_0) \setminus \{X\}. \square$$
Let $t_0 \in \mathcal{T}^{\mathbf{u}}$ be a subterm of a term $t = C[t_0] \in \mathcal{T}^{\mathbf{v}}$ identified by the *typed context* $C[\ ]^{\mathbf{u}}$. Given some other term $t_1 \in \mathcal{T}^{\mathbf{u}}$, if $C[t_1]$, i.e. the result of replacement of t_0 by t_1 in $C[\ ]^{\mathbf{u}}$, does not violate the convention of free variables in the definition of terms then, clearly, $C[t_1] \in \mathcal{T}^{\mathbf{v}}$. We say that a binary relation R on terms is *closed under typed contexts*, if
$$t_1\ R\ t_2 \text{ and } C[t_1] \in \mathcal{T}^{\mathbf{v}} \text{ implies } C[t_2] \in \mathcal{T}^{\mathbf{v}} \text{ and } C[t_1]\ R\ C[t_2].$$
Reducibility. The *immediate reducibility* of terms being the relation
$$(\lambda X^{\mathbf{u}}.\, t_1\ t_2) \rightarrow_\beta t_1[t_2/X]$$
between terms $t = (\lambda X^{\mathbf{u}}.\, t_1\ t_2)$ and $t_0 = t_1[t_2/X]$, $t, t_0 \in \mathcal{T}^{\mathbf{v}}$, such that $\lambda X^{\mathbf{u}}.\, t_1 \in \mathcal{T}^{(\mathbf{u} \rightarrow \mathbf{v})}$, $t_2 \in \mathcal{T}^{\mathbf{u}_1}$ for some $\mathbf{u}_1 \preceq \mathbf{u}$, and $X \in \mathbf{V}^{\mathbf{u}} \cap FV(t_1)$, the *reducibility relation* $\twoheadrightarrow$ is the least preorder containing $\rightarrow_\beta$ and closed under typed contexts $C[\ ]^{\mathbf{u}}$ and renaming of bound variables. A term t_0 is a *normal form* of a term t if $t \twoheadrightarrow t_0$ and t_0 is $\twoheadrightarrow$-minimal.

By the classical Church-Rosser theorem (see (Barendregt 1981)), the reducibility $\twoheadrightarrow$ is confluent and terminal. Then it is not difficult to prove that each term $t \in \mathcal{T}^{\mathbf{u}}$ has a unique normal form $t_0 : t \twoheadrightarrow t_0$ and $t_0 \in \mathcal{T}^{\mathbf{u}}$.

1.5 Discussion

The fundamental difference between logical and cognitive meanings is in the way the adnominal/adverbial modifiers are treated. The classical way (which can be traced back to Aristotle) is to interpret them as properties or, better to say, as conjunctive constraints to the intention of the modified unit: $\|(mod\ U)\| = \|mod\| \wedge \|U\|$. This is why, they obtain functional recursive types $(T/T), (T\backslash T)$, which seem to be sufficient to express the iterative types of $\mathcal{MT}^{\mathbf{lt}}$. But they only seem to. Through the Curry-Howard isomorphism, the meanings in the conventional system are isomorphic to derivations in formal systems, in which implication elimination corresponds to the function application and implication introduction corresponds to Lambda-abstraction. This leads directly to the Lambek calculus and its generalizations. However, it seems difficult to reconstruct this traditional semantic system from the facts of mother tongue acquisition in young children.

These facts (see Gleitman and Liberman 1995; Hirsh-Pasek and Golinkoff 1996) show that mother tongue is for the most part acquired by $3\frac{1}{2}$ year olds. Meanwhile, the psychologists witness that the children of this age are not capable of logical reasoning and do not perceive the world in terms of properties and relations. Their reasoning is taxonomic: the children distinguish between the general and the particular. Their perception is syncretic, i.e. they perceive integrally the eclectic combinations of heterogeneous entities and features united just by their juxtaposition. The toddlers under 18 months of age recognize nouns, adjectives, adverbs and verbs. Around this age they become biased to the rigid standard word order recognition. Due to this rigidity, they gradually create phrase- and then clause-sized units, which they parse in the same positions as those of the four initial categories above. So the initial primitive types $\mathbf{n}$, $\mathbf{q}$, $\mathbf{c}$ and $\mathbf{s}$ are formed. At about the same period, first instances of these types are developed. By the end of their second year of life, children need to attend to the verb's arguments in order to distinguish between causal and non-causal verbs, using the rigid word order. If we try to find the simplest grammatical model of the syntax gradually developed by the children of this age, we obtain the following universal archetype grammar, which is the source of our type system.

$$\begin{array}{ccl}
\mathbf{s} & \rightarrow & \underline{\mathbf{verb}} \oplus \{\mathbf{arg} \oplus\}^i\, \mathbf{c} \quad (1 \leq i \leq 5) \\
\mathbf{arg} & \rightarrow & \mathbf{n} \,|\, \mathbf{q} \,|\, \mathbf{c} \\
\mathbf{n} & \rightarrow & \mathbf{q} \oplus \underline{\mathbf{n}} \,|\, \mathbf{name} \\
\mathbf{q} & \rightarrow & \mathbf{c} \oplus \underline{\mathbf{q}} \,|\, \mathbf{adj} \\
\mathbf{c} & \rightarrow & \mathbf{c} \oplus \underline{\mathbf{c}} \,|\, \mathbf{adv}
\end{array}$$

($\oplus$ stands for the standard word order, which is a language dependent parameter). The first argument structures represented by this grammar serve for decomposing the parsed speech into primitive clause internal propositions with arguments (future situations) and simultaneously for carving the observed world into event sequences. Starting from the age of two years, the children become biased to morphological and syntactic cues. Their sentence comprehension becomes less dependent on events being described. As a result, the argument structures are abstracted into functions and become situations. Children start producing three-four word combinations (e.g., *I ride horsie*), in which for the first time they use all verb arguments together, i.e. they apply situations to their arguments. About the same period, they start using in their own production the pronouns (such as 'IT' or 'ONE') co-referential with full NP, i.e. they use context. But then the young children meet with a bottleneck preventing them from composing complex meaning of primitive situations. As we have explained it above, this ability needs planning. The acquisition of situation planning takes them several months. The first evidences of the use of type converters may be observed starting from the age of 26-27 months. Cf. the sentence *'See marching bear go'* produced by a 27 months old boy, in which three situations are combined by means of the diathesis $\mathbf{dth_3}(\mathbf{AGT}(1)^{\mathbf{n_a}})^{\mathbf{n}}\,\mathbf{EVT}$ applied to the situation $\mathbf{go_1}$ and the abstractor $\mathbf{abs}^x$ applied to the situation $\mathbf{march}$. The explosion of spontaneous speech by the age of $3 - 3\frac{1}{2}$ years is the evidence that the core of linguistic meaning is acquired at this age.

We think that the cognitive linguistic meaning plays the role of interface between syntax and reasoning mechanisms. The link with syntax is immediate. Choosing in the archetype grammar the heads the way they are underlined, we determine the semantic dependencies from functions to arguments, which have the same orientation as the corresponding surface syntax dependencies[6]. As to the interface with reasoning mechanisms, the logical meaning itself, deprived of relational structure and of entailment can serve for periphrasis, for text generation,

[6]If we do it for logical types of modifiers, we obtain semantic dependencies conflicting with the syntactic ones. Natural dependencies are simulated using multi-modal proofs.

but not for propositional attitude simulation. We think that it is more appropriate from the linguistic point of view to define logical semantics (be it type logical, or game-theoretic, or DRT, etc.) on top of the cognitive meanings that already have established references to the context, than to do it directly from sentences or from their syntactic structures.

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APPENDIX: Set Theoretic Semantics of $\mathcal{MT}^{\mathbf{lt}}$

Domains. For each primitive type $\mathbf{u} \in \mathbf{P}$, let a domain $D^{\mathbf{u}}$ be chosen so that the following conditions were met:
$(\mathbf{d_1})$ $D^{\perp} = \{\varepsilon\}$ for some object ε.
$(\mathbf{d_2})$ If $\mathbf{u} \prec \mathbf{v}$, then $D^{\mathbf{u}} \subset D^{\mathbf{v}}$.
$(\mathbf{d_3})$ $D^{\top} = \bigcup\limits_{\mathbf{u} \prec \top} D^{\mathbf{u}}$.

For each option type $\mathbf{u}^{(\mathbf{0})}$, a *null element* $\epsilon^{\mathbf{u}^{(\mathbf{0})}}$ of type $\mathbf{u}^{(\mathbf{0})}$ is selected and for each iterative type $\mathbf{u}^{(\omega)}$, a special object $[\,]^{\mathbf{u}^{(\omega)}}$ is selected called *empty list of type* $\mathbf{u}^{(\omega)}$, and the domains of option and iterative types are defined by:
$(\mathbf{d_4})$ If $\mathbf{u}^{(\mathbf{0})} \in \mathbf{O}$, then $D^{\mathbf{u}^{(\mathbf{0})}} = D^{\mathbf{u}} \cup \{\epsilon^{\mathbf{u}^{(\mathbf{0})}}\}$.
$(\mathbf{d_5})$ If $\mathbf{u}^{(\omega)} \in \mathbf{I}$, then $D^{\mathbf{u}^{(\omega)}} = \mathbf{list}(D^{\mathbf{u}})$, where $\mathbf{list}(D^{\mathbf{u}})$ is the set of all finite lists of objects in $D^{\mathbf{u}}$ with $[\,]^{\mathbf{u}^{(\omega)}}$ being the empty list. Somewhat more precisely, $\mathbf{list}(D^{\mathbf{u}})$ is the least set L containing $[\,]^{\mathbf{u}^{(\omega)}}$ and containing the pair $[e|l]$ for any $e \in L$ and $l \in L$.

$(\mathbf{d_6})$ For any $\mathbf{u}, \mathbf{v} \in \mathbf{B} \setminus \{\bot, \top\}$, if $\mathbf{u} \prec \mathbf{v}$, then $\epsilon^{\mathbf{u}^{(\mathbf{0})}} = \epsilon^{\mathbf{v}^{(\mathbf{0})}}$ and $[\,]^{\mathbf{u}^{(\omega)}} = [\,]^{\mathbf{v}^{(\omega)}}$.
$(\mathbf{d_7})$ If $(\mathbf{u} \to \mathbf{v}) \in \mathbf{T} \setminus \mathbf{B}$ and domains $D^{\mathbf{u}}$, $D^{\mathbf{v}}$ are defined, then the domain $D^{(\mathbf{u} \to \mathbf{v})}$ is defined as the set $(D^{\mathbf{v}})^{(D^{\mathbf{u}})}$ of all total functions from $D^{\mathbf{u}}$ to $D^{\mathbf{v}}$.

Interpretations. An *interpretation* $\iota = (\sigma, \tau, \omega, \Xi)$ consisting of *variables assignment* σ, *constants assignment* τ, *converters assignment* ω, and *contexts assignment* Ξ, is defined as follows:
$(\mathbf{i_1})$ σ is a total function from $\bigcup\limits_{\mathbf{u} \in (\mathbf{B} \setminus \{\top\})} \mathbf{V}^{\mathbf{u}}$ to $D^{\top}$ such that $\sigma(X) \in D^{\mathbf{u}}$ for $X \in \mathbf{V}^{\mathbf{u}}$.
$(\mathbf{i_2})$ τ is a total function from constants to objects of corresponding types (i.e. $\tau(c) \in D^{\mathbf{v}}$ for $c \in \mathbf{C}^{\mathbf{v}}$).
$(\mathbf{i_3})$ ω is a total function from converter constants to second order operators transforming $\mathbf{T}$-type functions to $\mathbf{T}$-type functions such that for each $\gamma \in \Omega^{(\mathbf{u} \Rightarrow \mathbf{v})}$, $\omega(\gamma) \in (D^{\mathbf{v}})^{(D^{\mathbf{u}})}$.
$(\mathbf{i_4})$ Ξ is a total function from terms, context operators and typed context variables into objects in $D^{\top}$: $\Xi(t, \phi, x) \in D^{\mathbf{u}}$ for any term t, context operator $\phi \in \mathbf{\Phi}$ and context variable $x \in \mathbf{R}^{\mathbf{u}}$.

For an interpretation $\iota = (\sigma, \tau, \omega, \Xi)$, an object variable $X \in \mathbf{V}^{\mathbf{u}}$ and an object $d \in D^{\mathbf{u}}$, we denote by $\iota\{X := d\}$ the interpretation $(\sigma_1, \tau, \omega, \Xi)$, in which $\sigma_1(Y) = \sigma(Y)$ for all $Y \neq X$ and
$\sigma_1(X) = d$.

Term values. For an *interpretation* $\iota = (\sigma, \tau, \omega, \Xi)$ and a term $t_0 \in \mathcal{T}^{\mathbf{u}}$, the *value* of t_0 in interpretation ι is denoted $||t_0||^{\mathbf{u},\iota}$ and defined by $||t_0||^{\mathbf{u},\iota} =_{df} ||t_0||^{t_0,\mathbf{u},\iota}$, where the value $||t||^{t_0,\mathbf{u},\iota}$ of sub-terms t of term t_0 is defined recursively as follows:
$(\mathbf{s_0})$ $||t||^{t_0,\mathbf{u},\iota} = \sigma(X)$ for $t = X \in \mathbf{V}^{\mathbf{u}}$.
$(\mathbf{s_1})$ $||t||^{t_0,\mathbf{u},\iota} = \tau(\mathbf{k})$ for $t = \mathbf{k} \in C^{\mathbf{u}}$.
$(\mathbf{s_2})$ Let $t = \phi_x$ for some $\phi \in \mathbf{\Phi}$ and $x \in \mathbf{R}^{\mathbf{u}}$. Then $||t||^{t_0,\mathbf{u},\iota} = \Xi(t_0, \phi, x)$.
$(\mathbf{s_3})$ Let $\mathbf{u} = \mathbf{v}^{(\mathbf{0})} \in \mathbf{O}$. Then $||t||^{t_0,\mathbf{u},\iota} = \varepsilon^{\mathbf{v}^{(\mathbf{0})}}$ if $t = \emptyset^{\mathbf{v}^{(\mathbf{0})}}$, and $||t||^{t_0,\mathbf{u},\iota} = ||t||^{t_0,\mathbf{v},\iota}$ if $t \neq \emptyset^{\mathbf{v}^{(\mathbf{0})}}$.
$(\mathbf{s_4})$ Let $\mathbf{u} = \mathbf{v}^{(\omega)}$ and $\mathbf{v}^{(\omega)} \in \mathbf{I}$. Then
(i) $||t||^{t_0,\mathbf{u},\iota} = [\,]^{\mathbf{v}^{(\omega)}}$, if $t = \mathbf{nil}^{\mathbf{v}^{(\omega)}}$,
(ii) $||t||^{t_0,\mathbf{u},\iota} = [||t_1||^{t_0,\mathbf{v}_1,\iota}, ..., ||t_k||^{t_0,\mathbf{v}_k,\iota}]$, if $t = [t_1, ..., t_k]$ and $t_i \in \mathcal{T}^{\mathbf{v}_i}$, for some $\mathbf{v}_i \prec \mathbf{v}$ and all $1 \leq i \leq k$.
$(\mathbf{s_5})$ $||t||^{t_0,\mathbf{u},\iota} = (\omega(\gamma))(||t_1||^{t_0,\mathbf{u},\iota})$, if $t = \gamma\{t_1\}$.
$(\mathbf{s_6})$ Let $t = (t_1\ t_2)$, where $t_1 \in \mathcal{T}^{(\mathbf{u}_1 \to \mathbf{u})}$ and $t_2 \in \mathcal{T}^{\mathbf{v}_1}$ for some $\mathbf{v}_1 \preceq \mathbf{u}_1$. Then

$$||t||^{t_0,\mathbf{u},\iota} = ||t_1||^{t_0,(\mathbf{u}_1 \to \mathbf{v}),\iota}(||t_2||^{t_0,\mathbf{v}_1,\iota}).$$

$(\mathbf{s_7})$ Let $\mathbf{u} = (\mathbf{u}_1 \to \mathbf{v})$, $t = \lambda X.\ t_1 \in \mathcal{T}^{(\mathbf{u}_1 \to \mathbf{v})}$ for some $t_1 \in \mathcal{T}^{\mathbf{v}}$ and $X \in \mathbf{V}^{\mathbf{u}_1} \cap FV(t_1)$. Then $||t||^{t_0,\mathbf{u},\iota} = f$, where f is the function defined by:

$$f(d) = ||t_1||^{t_0,\mathbf{u},\iota\{X:=d\}} \text{ for each } d \in D^{\mathbf{u}_1}. \ \Box$$

It is not difficult to prove that this definition is correct.

Proposition 2 $||t||^{\mathbf{u},\iota} \in D^{\mathbf{u}}$ *for any type* $\mathbf{u}$, *any term* $t \in \mathcal{T}^{\mathbf{u}}$ *and any interpretation* ι.

Besides this, term values are invariant with respect to reducibility.

Proposition 3 *If* $t_1 \twoheadrightarrow t_2$, *then* $||t_1||^{\mathbf{u},\iota} = ||t_2||^{\mathbf{u},\iota}$ *for any type* $\mathbf{u}$ *and interpretation* ι.