On Learning Discontinuous Dependencies from Positive Data

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Gold's Model : identification in the limit

- Algorithm:
  Input : a finite set of sentences (positive examples)
  Output : a grammar in the class that generates the sentences ; *the algorithm is required to converge*
- Formally:
  - $\mathcal{G}$: class of grammars
  - Σ : alphabet
  - ϕ : function from finite subsets of Σ^* to $\mathcal{G}$ such that
    $\forall G \in \mathcal{G}$, $\forall \langle e_i \rangle_{i \in \mathbb{N}}$ with $\mathcal{L}(G) = \langle e_i \rangle_{i \in \mathbb{N}}$:
    $\exists G' \in \mathcal{G}$ with $\mathcal{L}(G') = \mathcal{L}(G)$
    $\exists n_0 \in N$: $\forall n > n_0$ $\phi(\{e_1, \ldots, e_n\}) = G' \in \mathcal{G}$
    where $\mathcal{L}(G)$ *denotes the language associated to* G

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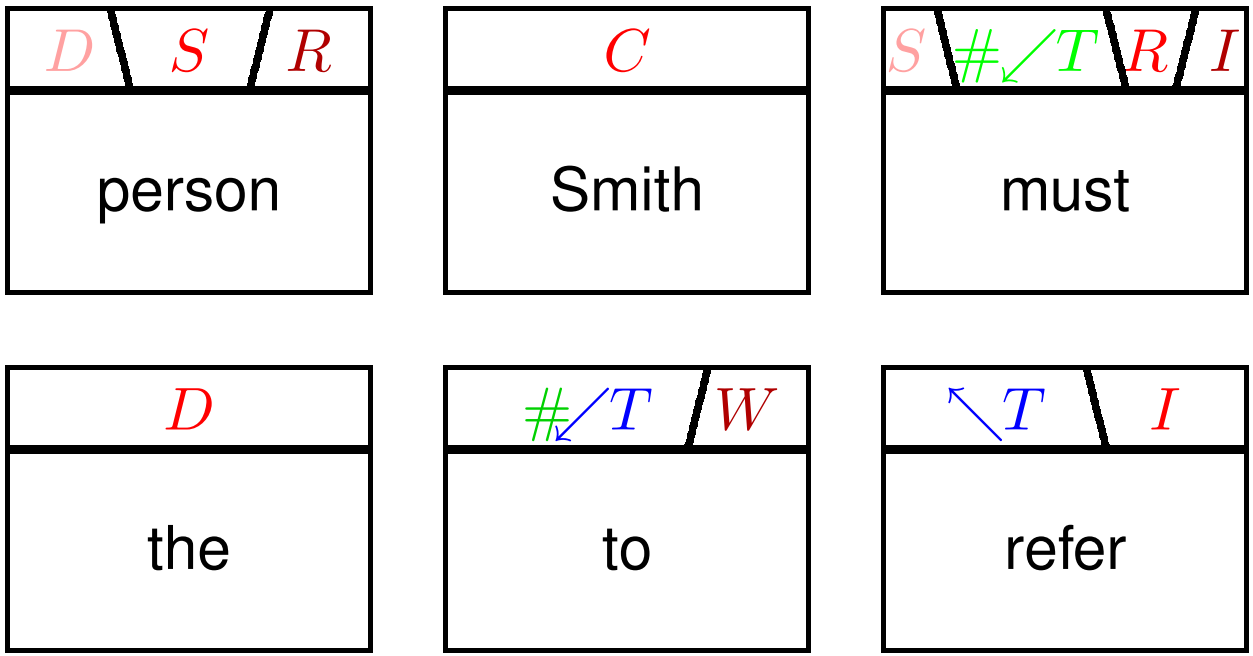
- Introduction
- Background [Dikovsky 2004]
  - Categories, dependencies, trees and nets
  - Categorial dependency grammars (CDGs)
  - Languages of (untyped) nets/depencency trees/strings
- Learning k-valued/rigid CDGs
  - Learning from untyped nets: an algorithm
  - Learning from untyped nets: finite elasticity
  - Learning from strings: finite elasticity (CDGs without optional or iterative types)
  - Non-Learnability for CDGs with optional or iterative types
- Conclusion

Categories, nodes, dependencies and nets

Categories and nodes:



$D = det$

$S = subj$

$R = attr\text{–}rel$

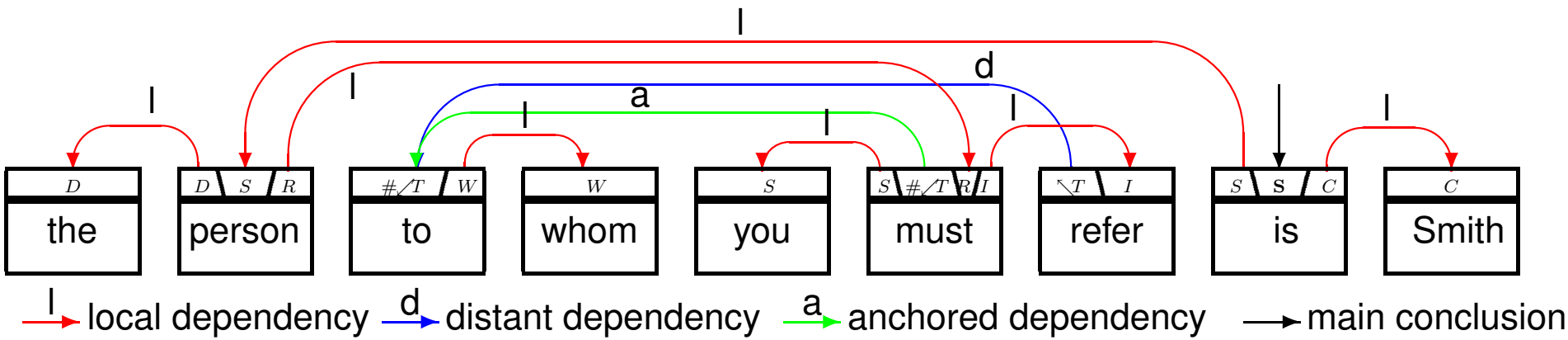
$C = n\text{–}copul$

$T = pre\text{–}TO\text{–}obj$

$W = prep\text{–}wh$

$I = inf\text{–}obj$

An untyped net:



CDG: definitions (1)

- Σ = alphabet for words in a natural language
- $\mathbf{C}$ = set of elementary categories : $det, subj, \cdots$
- $Start(\mathbf{C})$ and $End(\mathbf{C})$ = starting and ending atoms :
  - C : start or end of 1 local dependency
  - C^* : start of 0, 1 or more local dependencies (iterative)
  - C^+ : start of 1 or more local dependencies (repetitive)
  - $C^?$: start 0 or 1 local dependency (optional)
  - $\nwarrow C$: start (on the left) of 1 distant dependency
  - $\swarrow C$: end (from the right) of 1 distant dependency
  - $\#\swarrow C$: start of 1 anchored dependency or end of 1 anchored dependency and of 1 distant dependency
  - $\nearrow C$, $\searrow C$ and $\#\searrow C$ for the reverse direction

CDG: definitions (2)

- $Cat(\mathbf{C})$ = categories = $List(Start(\mathbf{C})) \times End(\mathbf{C}) \times List(Start(\mathbf{C}))$ written $L_1 \backslash \cdots \backslash L_i \backslash C / R_j / \cdots / R_1$
- $\vdash$ = provability relation based on simplification rules (local, distant and anchored dependencies):

$$
\begin{array}{lcll}
\Gamma_1 C[C\backslash\alpha]\Gamma_2 & \vdash & \Gamma_1\alpha\Gamma_2 & (\mathbf{L}) \\
\Gamma_1 C[C^*\backslash\alpha]\Gamma_2 & \vdash & \Gamma_1[C^*\backslash\alpha]\Gamma_2 & (\mathbf{I}) \\
\Gamma_1 C[C^+\backslash\alpha]\Gamma_2 & \vdash & \Gamma_1[C^*\backslash\alpha]\Gamma_2 & (\mathbf{R}) \\
\Gamma_1 C[C^?\backslash\alpha]\Gamma_2 & \vdash & \Gamma_1\alpha\Gamma_2 & (\mathbf{O}) \\
\Gamma_1[C^{?|*}\backslash\alpha]\Gamma_2 & \vdash & \Gamma_1\alpha\Gamma_2 & (\mathbf{\Omega}) \\
\Gamma_1\#(\alpha)[\#(\alpha)\backslash\beta]\Gamma_2 & \vdash & \Gamma_1\alpha\beta\Gamma_2 & (\mathbf{A}) \\
\Gamma_1(\swarrow C)\Gamma_2^{(1)}[(\nwarrow C)\backslash\alpha]\Gamma_3 & \vdash & \Gamma_1\Gamma_2\alpha\Gamma_3 & (\mathbf{D})
\end{array}
$$

(1) Γ_2 without $\swarrow C, \nwarrow C$ and $\#\swarrow C$

CDG: definitions (3)

- A *CDG* is a finite relation G between Σ and $Cat(\mathbf{C})$
- G *generates* a string $c_1 \ldots c_n \in \Sigma^+$ iff
  $\exists A_1, \ldots, A_n \in Cat(C)$ such that:
  $G : c_i \mapsto A_i \ (1 \leq i \leq n)$ and $A_1 \cdots A_n \vdash S$
- The language of G,
  $\mathcal{L}(G)$ = the set of strings generated by G
- G k-valued iff at most k types per symbol (rigid if $k = 1$)

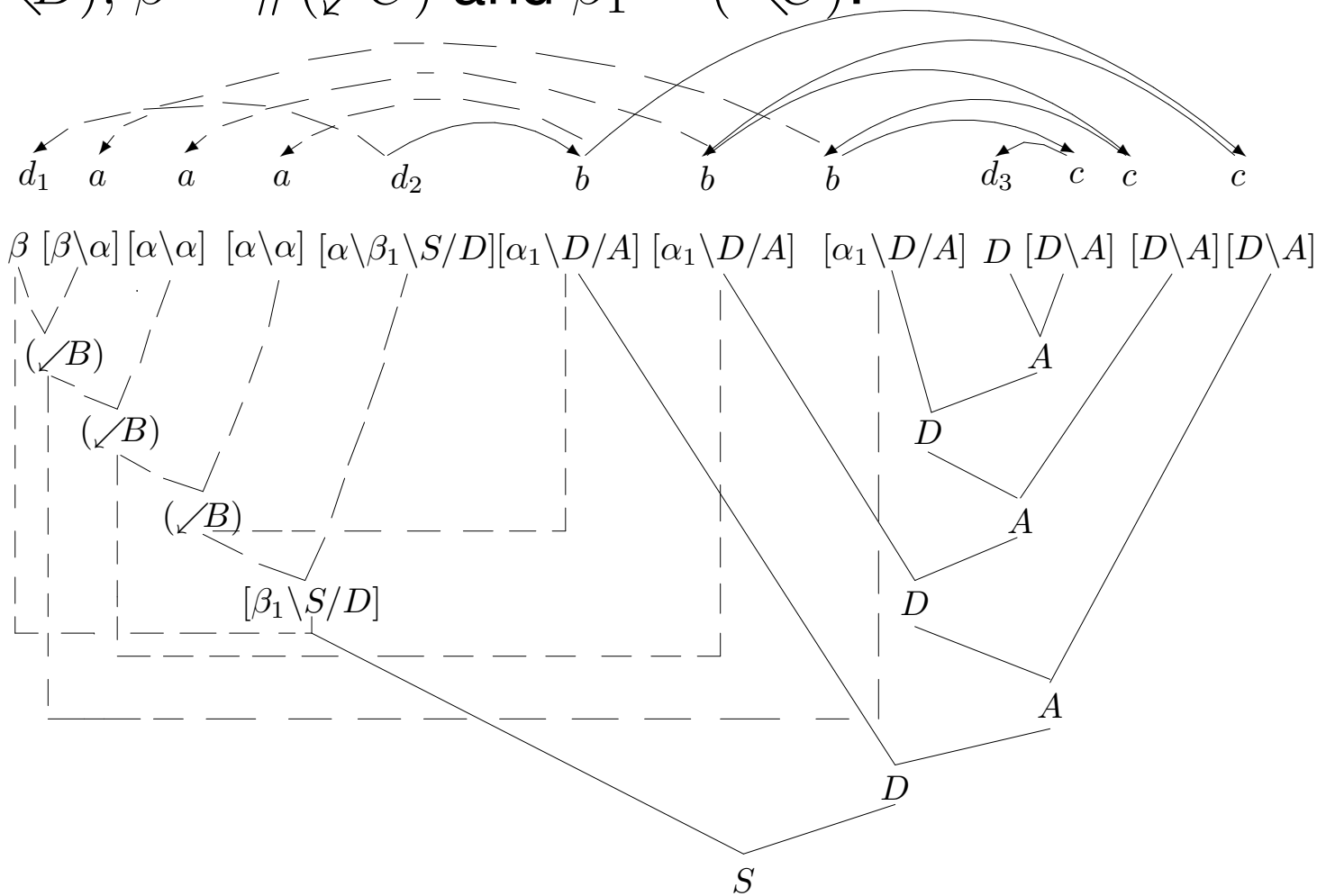
Example of a CDG (1)

$$G_0 : \begin{cases} a & \mapsto & \#\swarrow C \backslash \#\swarrow B \ \mid \ \#\swarrow B \backslash \#\swarrow B \\ b & \mapsto & \nwarrow B \backslash D \\ c & \mapsto & D \backslash A \\ d_1 & \mapsto & \#\swarrow B \\ d_2 & \mapsto & \#\swarrow B \backslash \nwarrow C \backslash S / D \\ d_3 & \mapsto & D \end{cases}$$

$\mathcal{L}(G_0) = \{d_1 a^n d_2 b^n d_3 c^n \mid n > 0\}$: a non-CF language

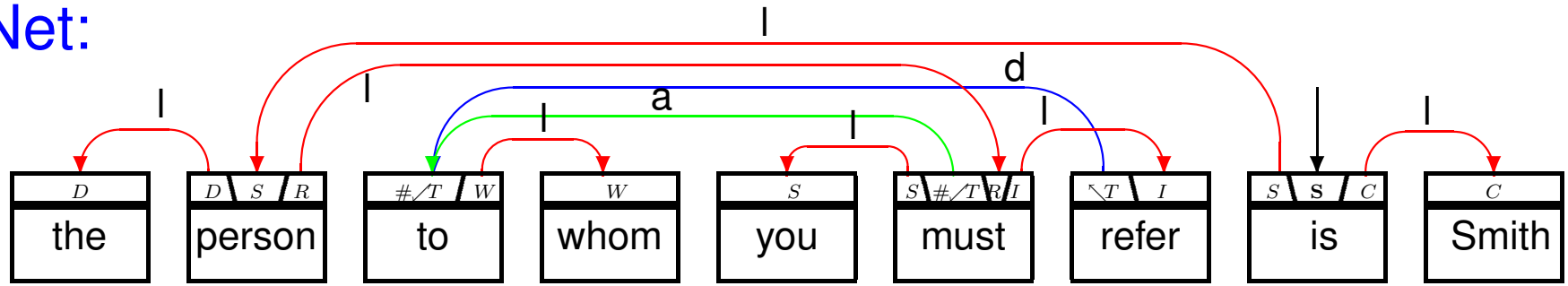
Example of a CDG (2)

A proof of $d_1a^3d_2b^3d_3c^3 \in L(G_0)$, in which $\alpha = \#(\swarrow B)$, $\alpha_1 = (\nwarrow B)$, $\beta = \#(\swarrow C)$ and $\beta_1 = (\nwarrow C)$:

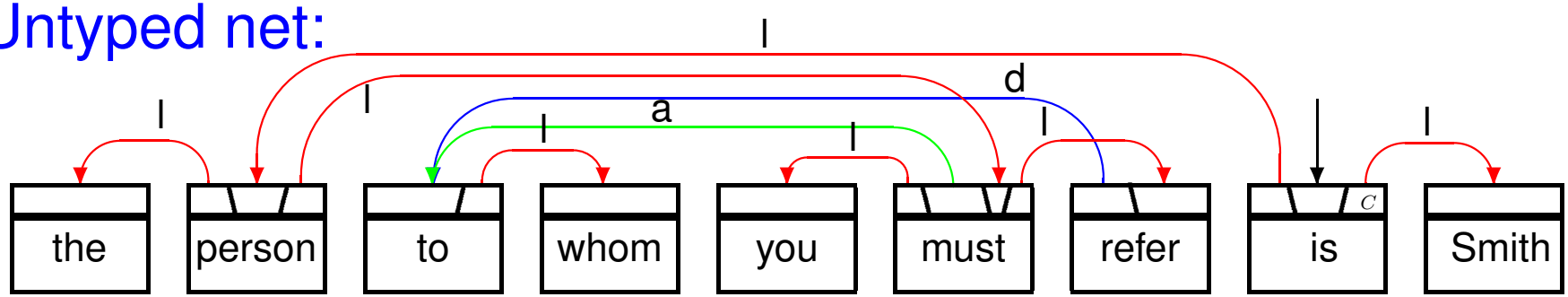


Net, untyped net, tree or string

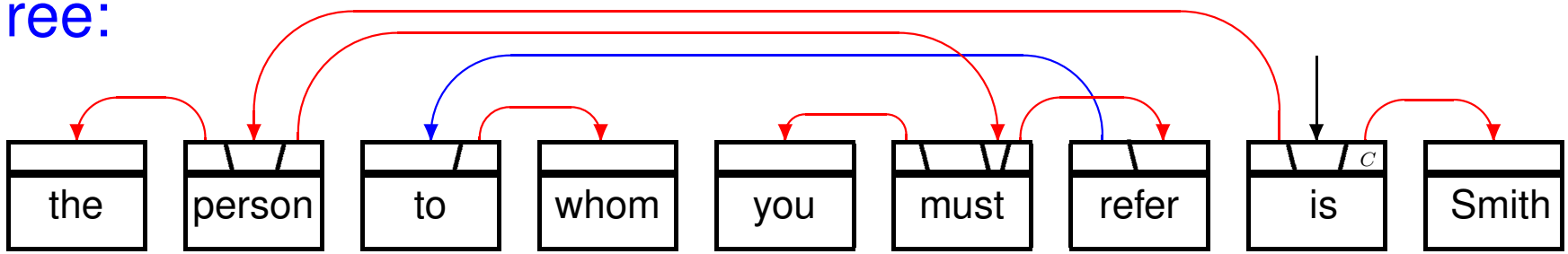
Net:



Untyped net:



Tree:



String:

the person to whom you must refer is Smith

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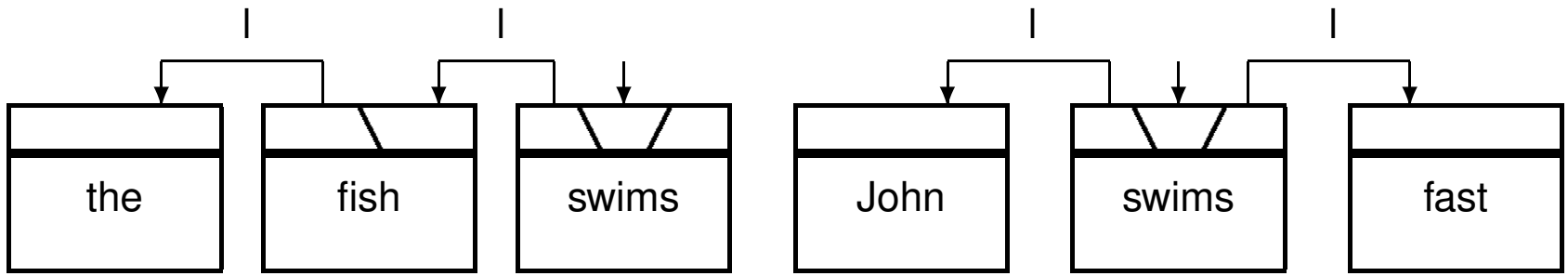
Learning from untyped nets

(Rigid grammars)

Starting with the following positive examples

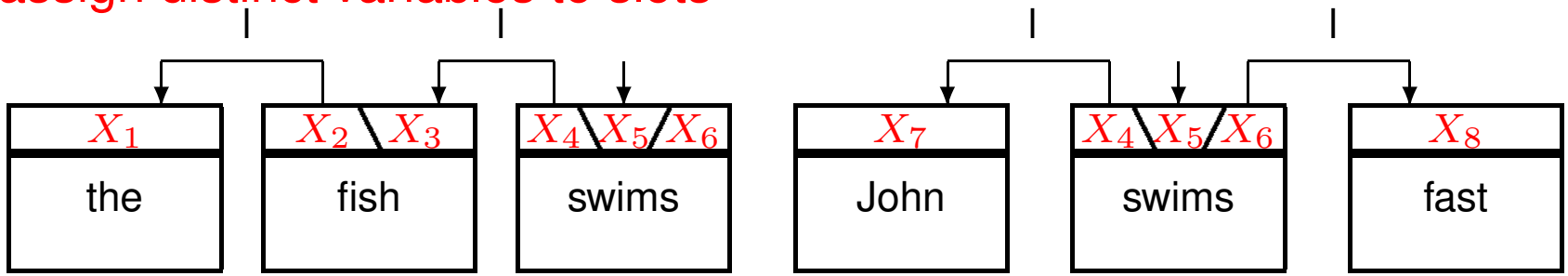


Learning from untyped nets

(Rigid grammars)

Starting with the following positive examples

we assign distinct variables to slots



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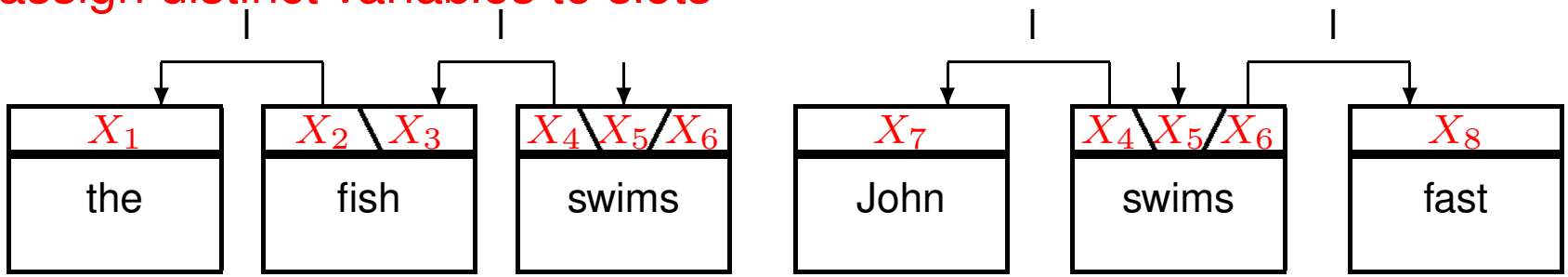

and obtain the resulting grammar after unification (and link counting)

	General form	Unification	Rigid grammar
the	X_1		X_1
$fish$	$X_2 \backslash X_3$	$X_2 = X_1$	$X_1 \backslash X_3$
$swims$	$X_4 \backslash X_5 / X_6$	$X_4 = X_3; X_5 = S$	$X_3 \backslash S / X_6^?$
$John$	X_7	$X_7 = X_4 (= X_3)$	X_3
$fast$	X_8	$X_8 = X_6$	X_6

Finite elasticity and learnability

Infinite elasticity:

$\exists (e_i)_{i \in \mathbb{N}}$ an infinite sequence of sentences
$\exists (L_i)_{i \in \mathbb{N}}$ an infinite sequence of languages

$$\text{such that } \begin{cases} \forall i \in \mathbb{N} \ : \ e_i \notin L_i \\ \{e_0, \ldots, e_{i-1}\} \subseteq L_i \end{cases}$$

Properties [Wright 1989]

- unlearnability $\Rightarrow$ infinite elasticity
- finite elasticity $\Rightarrow$ learnability

Rigid languages of untyped nets

Theorem:

1. Finite elasticity (in fact *bounded finite thickness*)
2. The class is learnable
3. The given algorithm learns the class

Proof schemes:

(1) There is a finite number of "reduced" languages in the class that correspond to a given list of untyped nets
(2) Consequence of finite elasticity
(3) The returned grammar corresponds to the minimal language compatible with the input untyped nets

Proof of finite elasticity (1)

[Shinohara 1990]: finite thickness $\Rightarrow$ finite elasticity

- **Definition.** $G_1 \sqsubseteq G_2$ iff
  $\forall x \in \Sigma, C \in Cat(\mathbf{C}), G_1 : x \mapsto C \Rightarrow G_2 : x \mapsto C$
- **Definition and lemma.** The mapping $\mathcal{L}_{UNet}$ from link grammars to untyped net languages is monotonic: if $G_1 \sqsubseteq G_2$ then $\mathcal{L}_{UNet}(G_1) \subseteq \mathcal{L}_{UNet}(G_2)$
- **Definition.** A grammar G is reduced with respect to a set X of untyped nets iff $X \subseteq \mathcal{L}_{UNet}(G)$ and for each grammar $G' \sqsubseteq G$, $G' \neq G \Rightarrow X \not\subseteq \mathcal{L}_{UNet}(G')$

Proof of finite elasticity (2)

- **Lemma.** For each finite set $X \subseteq \mathcal{L}_{UNet}(G)$, there is a finite set of rigid untyped net languages that correspond to the grammars that are reduced from X
- **Definition.** Monotonicity and the previous property define a system that has bounded finite thickness
- **Theorem [Shinohara 1990].** A formal system that has bounded finite thickness has finite elasticity
- **Corollary.** Rigid untyped net languages have finite elasticity

k-valued languages of strings (1)

Theorem [Kanasawa 1998]:

The images, through a finite-valued relation of a class of languages that has finite elasticity, forms a class that has finite elasicity

Consequences:

1. For each k, the class of k-valued CDG (without optional and iterative category) languages of strings has finite elasticity
2. For each k, this class (of CDGs) is learnable from strings

k-valued languages of strings (2)

Lemma:

1. For each k, the classes of k-valued CDG without optional and/or iterative category languages of strings accept a limit point
2. For each k, these classes (of CDGs) are not learnable from strings

Conclusion

- Rigid CDGs are learnable from untyped nets
- There exists a simple learning algorithm for this class
- Theoretical learnability result (finite elasticity) for k-valued CDGs without optional and iterative category learned from strings
- Rigid (or k-valued) CDGs with optional and/or iterative category are not learnable from strings

Class	Learnable from strings		Finite elasticity on strings		Finite elasticity on structures		Finite-valued relation
A^*	no	$\Rightarrow$	no		yes	$\Rightarrow$	no
$A^?$	no	$\Rightarrow$	no		yes	$\Rightarrow$	no
A^+	yes	$\Leftarrow$	yes	$\Leftarrow$	yes		yes