

Categorial Dependency Grammars

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Categorial Grammars
Grammaires Catégorielles

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Abstract : *Categorical Dependency Grammars* are proposed as a model of word-driven (in contrast with head-driven) dependency syntax.

1. CDG : a subcommutative resource-sensitive calculus of dependency types,
2. enough expressive to naturally treat surface syntax phenomena including discontinuous constructions,
3. strongly more expressive than link grammars and many other projective DGs,
4. incomparable with multi-TAGs,
5. parsed in polynomial time (Earley-type parser),
6. disposing of an under-specified formal semantics,
7. learnable from strings, when rigid typed.

PLAN :

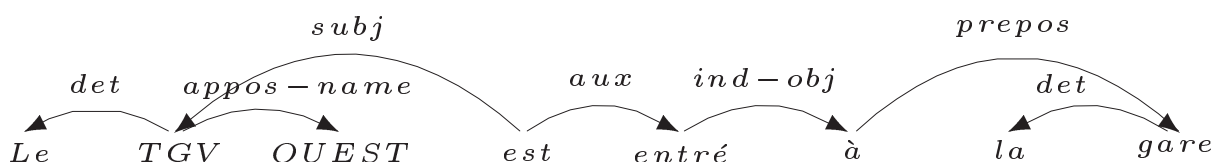
- I. INTRODUCTION :
WORD-DRIVEN DEPENDENCIES
- II. Categorical dependency grammars
- III. Expressivity
- IV. Complexity
- V. Problematic

I. Linguistic introduction

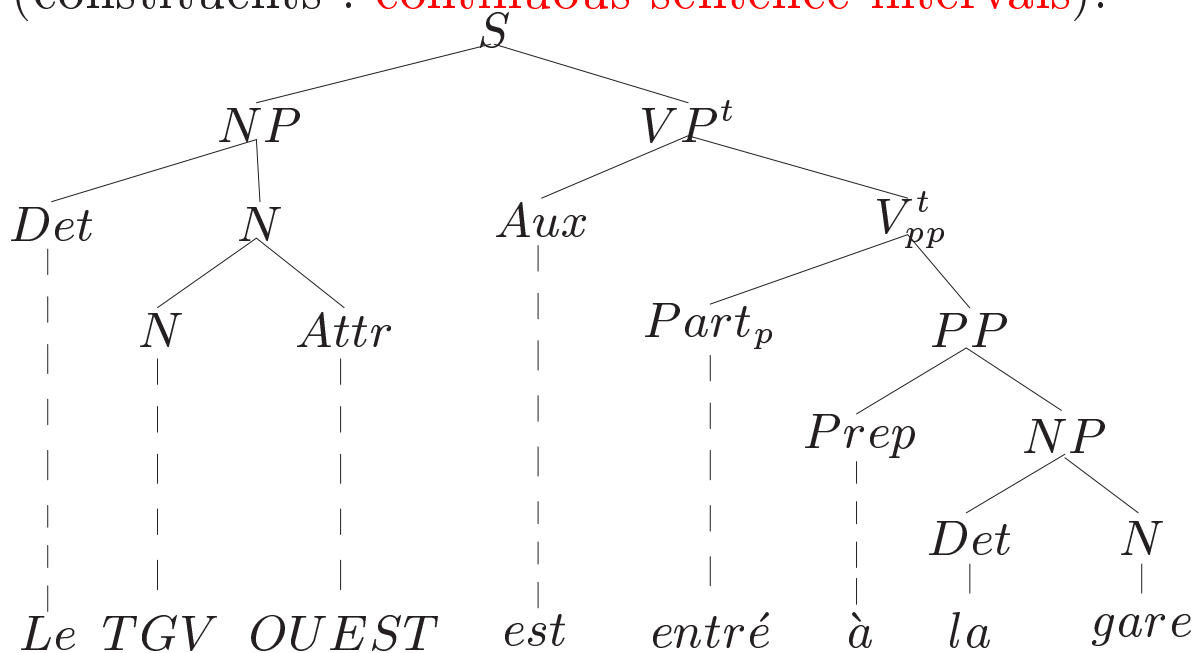
I.1 Syntactic dependencies

Two structural syntactic paradigms :

- **dependency trees** : (dependencies : irreflexive, anti-symmetric, anti-transitive **binary relations on words**).



- **constituent (syntagmatic) trees** : (constituents : **continuous sentence intervals**).



I.2. Head driven dependencies :

(Gladky'66, Robinson'70, Jackendoff'77)

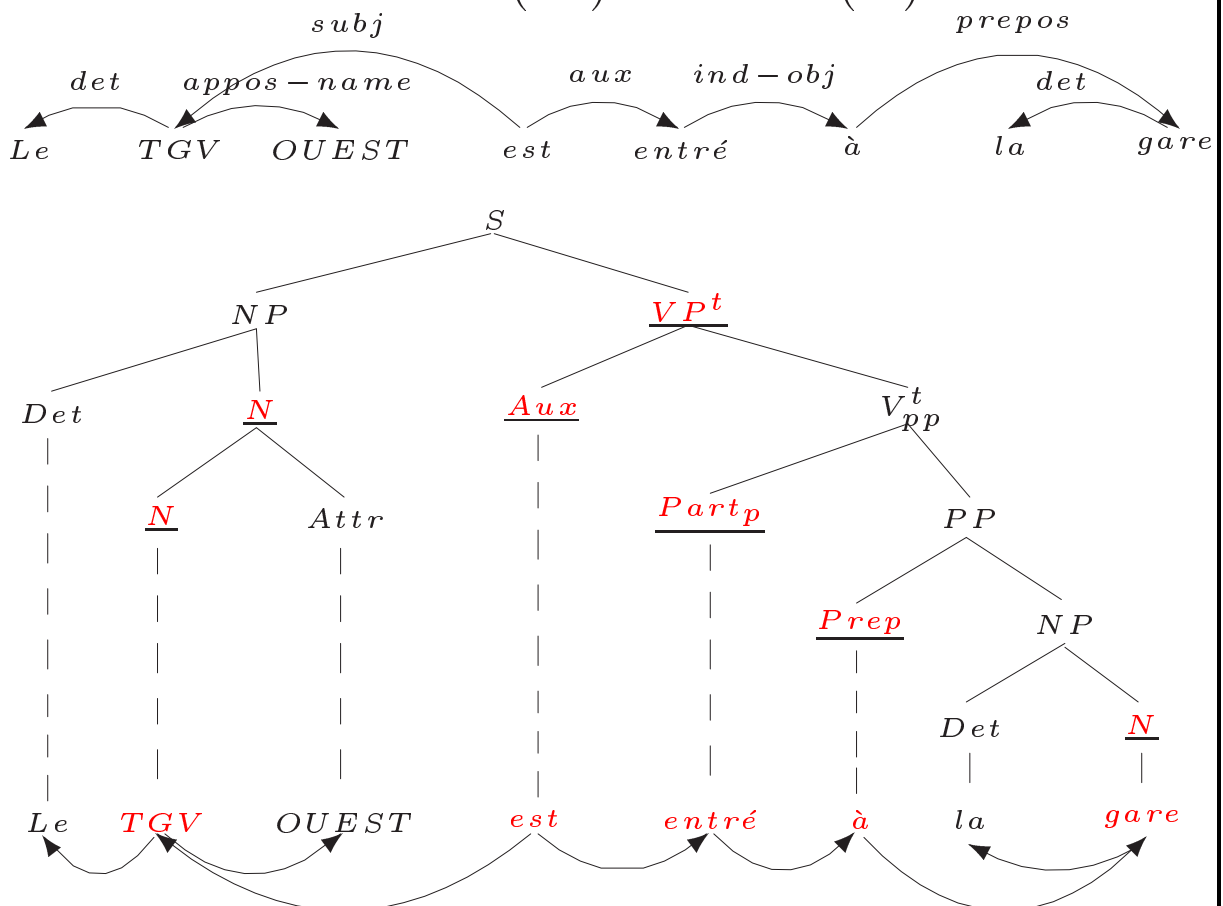
By-product of selection of a single **head** constituent in each non-unit constituent :

- a word w is the head of C :

$\forall C' (\{w\} \subseteq C' \subset C \Rightarrow \text{selected}(\underline{C}'))$

(C is **projection** of w).

- $C \subset C' \Rightarrow \text{head}(C') \Rightarrow^* \text{head}(C)$.



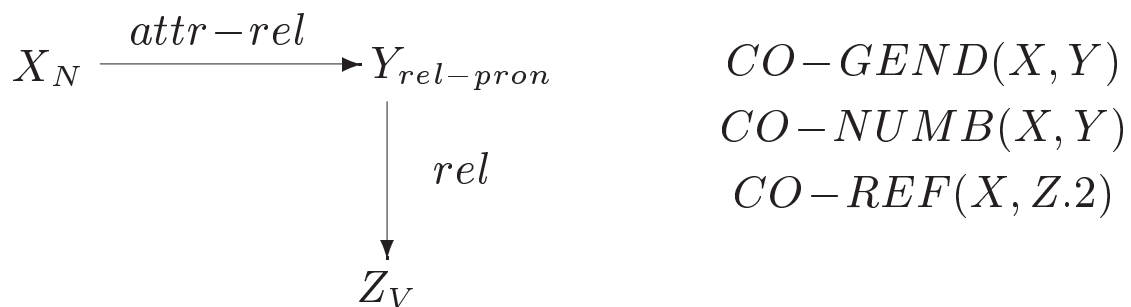
I.3 Word driven dependencies :

G (*overnor*) \xrightarrow{D} S (*ubordinate*) :

Binary relation D : $\ll G$ licences $S \gg$

verifying constraints on lexical and grammatical features, WO constraints, pronominalization constraints, etc. concerning **only** G and S .

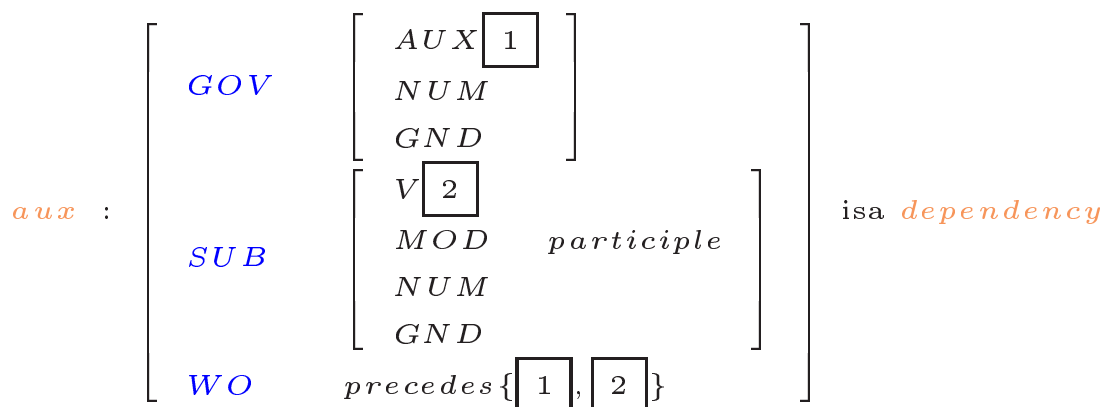
EX1 : (Mel'čuk'88)



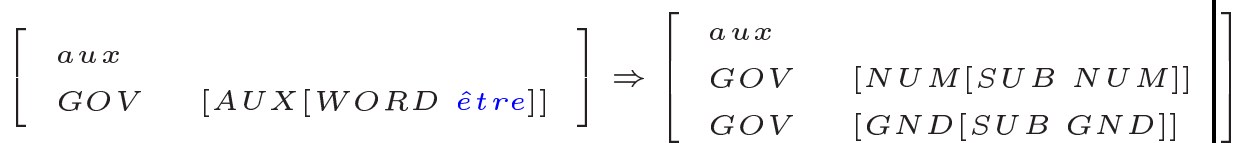
EX2 :

est \xrightarrow{aux} *entré*

is an instance of :



Agreement :



EX3 : Cf. (in the sentence *Le TGV ...*) :

entré $\xrightarrow{\text{ind-obj}}$ *à*

ind-obj :

<i>GOV</i>	<table style="border-collapse: collapse;"> <tr><td style="padding: 2px 10px;"><i>V</i></td></tr> <tr><td style="padding: 2px 10px;"><i>VAL</i> { 1, 2 }</td></tr> <tr><td style="padding: 2px 10px;"><i>SYN</i> [<i>¬modal</i>, <i>¬copul</i>]</td></tr> </table>	<i>V</i>	<i>VAL</i> { 1 , 2 }	<i>SYN</i> [<i>¬modal</i> , <i>¬copul</i>]	isa <i>dependency</i>
<i>V</i>					
<i>VAL</i> { 1 , 2 }					
<i>SYN</i> [<i>¬modal</i> , <i>¬copul</i>]					
<i>SUB</i>	[<i>WORD</i> 3 <i>preposition</i>]				
<i>WO</i>	<i>precedes</i> { 1 , 3 }				

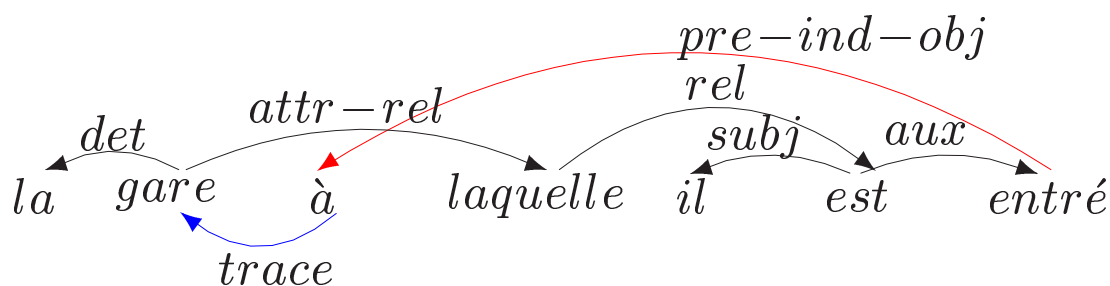
But also :

à $\xleftarrow{\text{pre-ind-obj}}$ *entré*

pre-ind-obj :

<i>GOV</i>	<table style="border-collapse: collapse;"> <tr><td style="padding: 2px 10px;"><i>V</i></td></tr> <tr><td style="padding: 2px 10px;"><i>VAL</i> { 1, 2 }</td></tr> <tr><td style="padding: 2px 10px;"><i>SYN</i> [<i>¬modal</i>, <i>¬copul</i>]</td></tr> </table>	<i>V</i>	<i>VAL</i> { 1 , 2 }	<i>SYN</i> [<i>¬modal</i> , <i>¬copul</i>]	isa <i>dependency</i>
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as in the sentence :

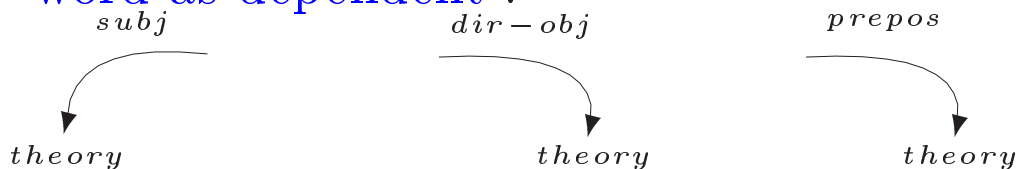


I.4. Types of word dependencies

Primitive types : **dependency names** (!!) (not constituent / constituent complement classes).

Complex types determine : oriented local valencies, polarized discontinuous valencies, order, adjacency :

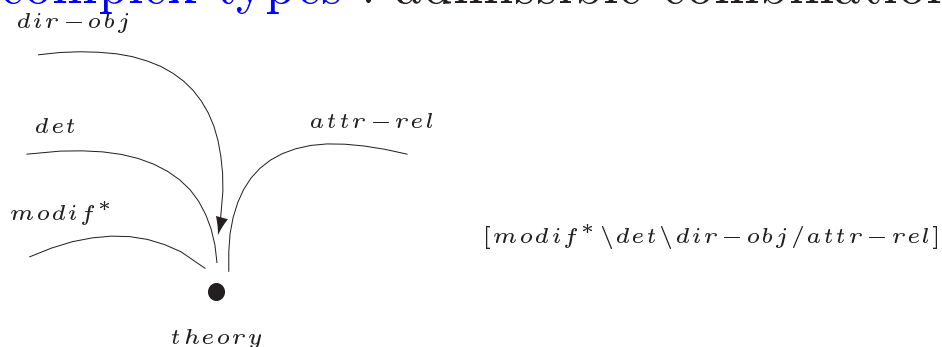
- **word as dependent** :



- **word as governor** :



- **complex types** : admissible combinations :



I.5. Definition of syntactic types (categories)

Polarities of distant dependencies :

Negative right : $\swarrow pre-indir-obj$: (expects right governor through the dependency $pre-indir-obj$),

Positive right : $\nearrow neg-compos$: (expects right subordinate through the dependency $neg-compos$).

Dual (\check{C}) : ($\swarrow pre-indir-obj$) et ($\nwarrow pre-indir-obj$), ($\nearrow neg-compos$) et ($\searrow neg-compos$).

Negative anchored (on a category) :

$\#(\swarrow clit-obj)$: clitic (e.g. anchored on a verb with category $[\#(\swarrow clit-obj)\backslash S/aux]$).

Negative not anchored are loose.

\mathbf{C} being a set of **primitive categories**,

$V^+(\mathbf{C}) =_{df} \{(\nwarrow C), (\nearrow C) \mid C \in \mathbf{C}\}$: positive,

$V^-(\mathbf{C}) =_{df} \{(\swarrow C), (\searrow C) \mid C \in \mathbf{C}\}$: negative,

$Anc(\mathbf{C}) =_{df} \{\#(\alpha) \mid \alpha \in V^-(\mathbf{C})\}$: anchored,

$\mathbf{C}^* =_{df} \{C^* \mid C \in \mathbf{C}\}$: iterated.

Definition of complex categories (1^{st} order) :

$Cat(\mathbf{C})$: minimal set such that :

1. $\mathbf{C} \cup V^-(\mathbf{C}) \cup Anc(\mathbf{C}) \subset Cat(\mathbf{C})$.
2. For $C \in Cat(\mathbf{C})$,

$A_1 \in (\mathbf{C} \cup \mathbf{C}^* \cup Anc(\mathbf{C}) \cup \swarrow \mathbf{C})$ et

$A_2 \in (\mathbf{C} \cup \mathbf{C}^* \cup Anc(\mathbf{C}) \cup \nearrow \mathbf{C})$

$[A_1 \setminus C], [C / A_2] \in Cat(\mathbf{C})$.

Constructors $\setminus, /$ are associative. So each category has the form :

$$[L_k \setminus \dots L_1 \setminus C / R_1 \dots / R_m].$$

EX : $[\#(\swarrow \textit{clit} - \textit{dobj}) \setminus \textit{subj} \setminus S / \textit{aux}]$

is one of the categories of the auxiliary *être*, which anchors a clitic and expects a subordinate subject on its left and expects a subordinate participle on its right (def. of **aux**).

II. Categorical dependency grammar (CDG)

$$G = (W, \mathbf{C}, S, \delta),$$

where W : words, \mathbf{C} : categories, $S \in \mathbf{C}$,

δ (**lexicon**) : finite substitution $W \rightarrow 2^{Cat(\mathbf{C})}$.

EX 4 : $les \mapsto det$,
 $bordellais \mapsto modif$,
 $excellents \mapsto cop-adj$,
 $vins \mapsto [det \setminus subj / modif^*]$,
 $sont \mapsto [subj \setminus S / cop-adj^*]$

D-form (Δ, Γ) of a sentence $w = a_1 \dots a_n$:

- Δ : a dependency graph (DG) on w ,
- Γ : a string of positioned categories of DGs,
 α^i : a category of a DG on w with the root a_i ,
- $((V, \emptyset), C_1^1 \dots C_n^n)$: **initial D-form**, if
 $C_i \in \delta(a_i)$, $1 \leq i \leq n$,

EX : $det^1 [det \setminus subj / modif^*]^2 modif^3 [subj \setminus S / cop-adj^*]^4 cop-adj^5$

- (Δ, S^j) : **terminal D-form**.

II.1. Sub-commutative dependency calculus

$$\mathbf{L}^1. ((V, E), \Gamma_1 C^j [C \setminus \beta]^l \Gamma_2) \vdash \\ ((V, E \cup \{a_j \xleftarrow{C} a_l\}), \Gamma_1 [\beta]^l \Gamma_2), \quad C \in \mathbf{C}.$$

$$\mathbf{I}^1. ((V, E), \Gamma_1 C^j [C^* \setminus \beta]^l \Gamma_2) \vdash \\ ((V, E \cup \{a_j \xleftarrow{C} a_l\}), \Gamma_1 [C^* \setminus \beta]^l \Gamma_2), \quad C \in \mathbf{C}.$$

$$\mathbf{\Omega}^1. ((V, E), \Gamma_1 [C^* \setminus \beta]^l \Gamma_2) \vdash ((V, E), \Gamma_1 \beta^l \Gamma_2).$$

$$\mathbf{A}^1. ((V, E), \Gamma_1 \#(\alpha)^j [\#(\alpha) \setminus \beta]^l \Gamma_2) \vdash \\ ((V, E), \Gamma_1 \alpha^j \beta^l \Gamma_2), \quad \#(\alpha) \in \mathit{Anc}(\mathbf{C}).$$

$$\mathbf{P}^1. ((V, E), \Gamma_1 [(\swarrow C) \setminus \alpha]^j \Gamma_2) \vdash \\ ((V, E), \Gamma_1 (\swarrow C)^j \alpha^j \Gamma_2).$$

$$\mathbf{C}^1. ((V, E), \Gamma_1 C^l \alpha^j \Gamma_2) \vdash ((V, E), \Gamma_1 \alpha^j C^l \Gamma_2) \\ \text{if } \alpha \in (\swarrow \mathbf{C} \cup \searrow \mathbf{C}) \text{ and } C \in \mathit{Cat}(\mathbf{C}) \text{ without} \\ \text{occurrences of } \alpha, \#(\alpha) \text{ and } \check{\alpha}.$$

$$\mathbf{D}^1. ((V, E), \Gamma_1 (\swarrow C)^j (\swarrow C)^l \Gamma_2) \vdash \\ ((V, E \cup \{a_j \xleftarrow{C} a_l\}), \Gamma_1 \Gamma_2).$$

II.2. EX 5 : Example of a derivation :

les \mapsto *det*,

bordellais \mapsto *modif*,

excellents \mapsto *cop-adj*,

vins \mapsto [*det*\ *subj*/*modif**],

sont \mapsto [*subj*\ *S*/*cop-adj**]

$det^1 [det \backslash subj / modif^*]^2 modif^3 [subj \backslash S / cop - adj^*]^4$

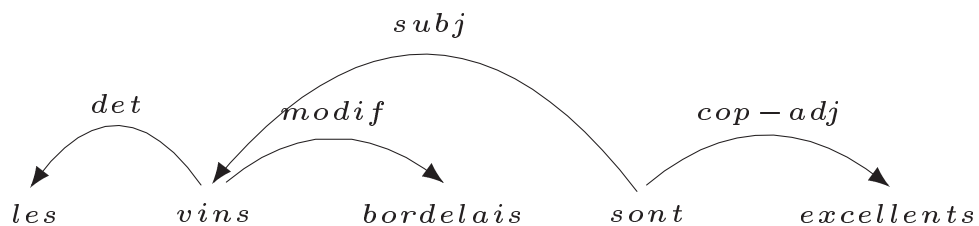
$cop - adj^5 \vdash$

$det^1 [det \backslash subj / modif^*]^2 [subj \backslash S / cop - adj^*]^4 cop - adj^5$

$det^1 [det \backslash subj / modif^*]^2 [subj \backslash S / cop - adj^*]^4 \vdash$

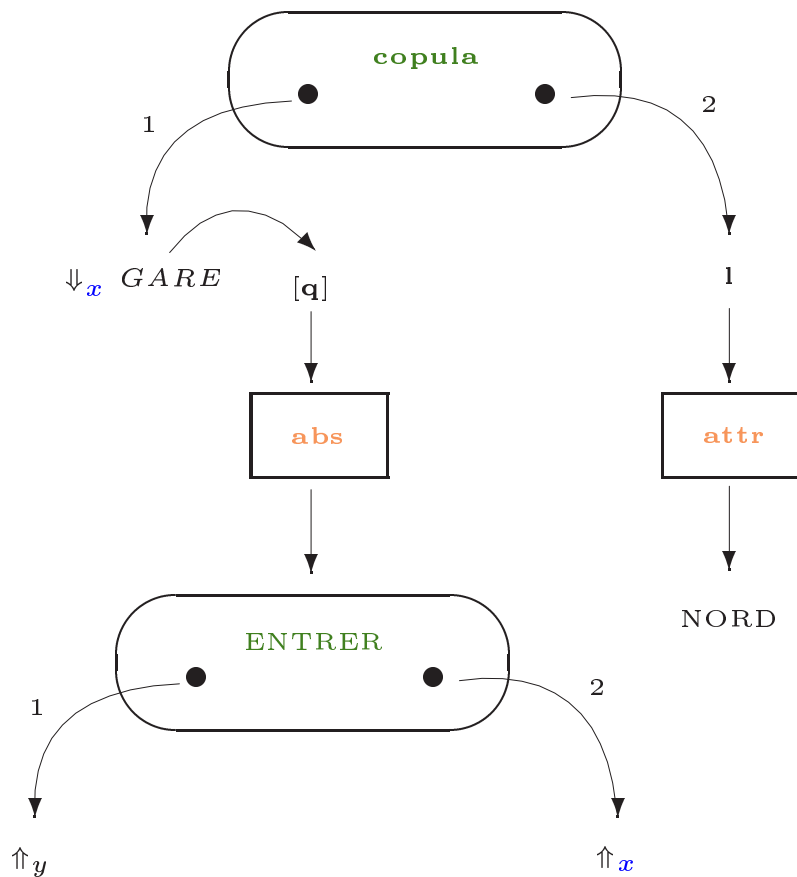
$det^1 [det \backslash subj]^2 [subj \backslash S / cop - adj^*]^4 \vdash$

$det^1 [det \backslash subj]^2 [subj \backslash S]^4 \vdash^* S^4$



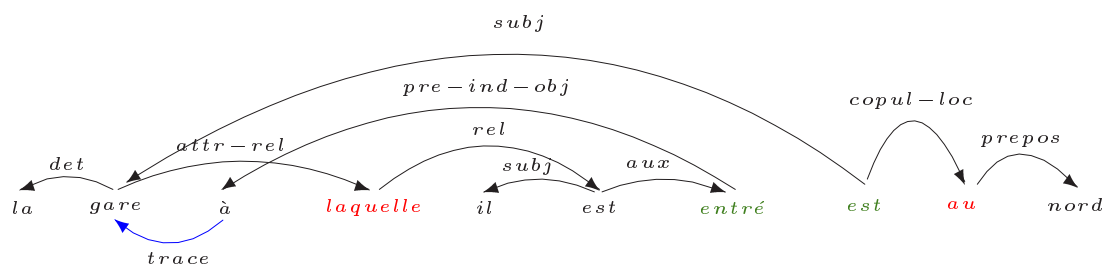
II.3. Types are justified by situational meaning

EX 6 :



See [A. Dikovsky, Formal Grammar'2003.]

The corresponding dependency structure :



II.4. Generated languages :

$G_C(D, w) : (\Delta_0, \Gamma_0) \vdash_C^* (D, S^j)$, for an initial D-form (Δ_0, Γ_0) of w and some j .
($C \in \{sc, FA\}$).

D-language of G :

$$D_C(G) =_{df} \{D \mid \exists w G_C(D, w)\}.$$

Language of G :

$$L_C(G) =_{df} \{w \mid \exists D G_C(D, w)\}.$$

Families :

$$\mathcal{D}(CDG^{sc}), \mathcal{L}(CDG^{sc}).$$

III. Expressivity

III.1. Projectivity / non-projectivity

Head-driven DTs are projective. Word-driven DTs may be non-projective :

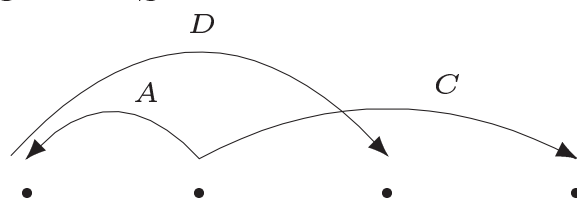
EX 7. Non-projective DTs :

$$[A/(\nearrow D)]^1 [A \setminus S/C / \#(\searrow D)]^2 \#(\searrow D)^3 C^4 \vdash$$

$$[A/(\nearrow D)]^1 [A \setminus S/C]^2 (\searrow D)^3 C^4 \vdash^*$$

$$A^1 [A \setminus S/C]^2 (\nearrow D)^1 (\searrow D)^3 C^4 \vdash$$

$$A^1 [A \setminus S/C]^2 C^4 \vdash^* S^2$$



III.2. Cycles in dependency structures :

EX 8 :

$$[(\swarrow C)/(\nearrow A)]^1 [(\nwarrow C) \setminus (\searrow A)]^2 S^3 \vdash$$

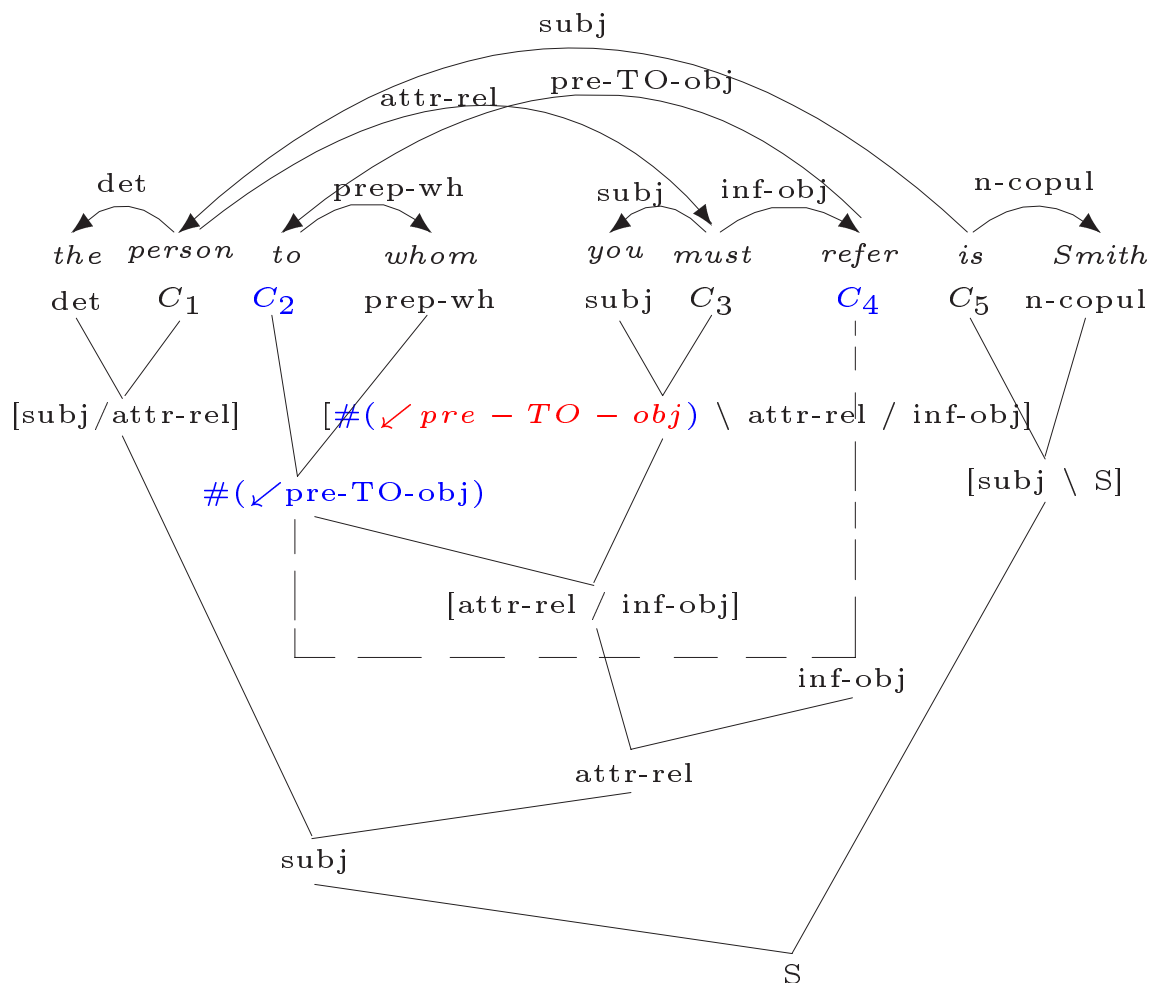
$$(\swarrow C)^1 (\nearrow A)^1 (\nwarrow C)^2 (\searrow A)^2 S^3 \vdash$$

$$(\swarrow C)^1 (\nwarrow C)^2 (\nearrow A)^1 (\searrow A)^2 S^3 \vdash^* S^3$$



III. Relative clauses

EX 9 :



person $\mapsto C_1 = [det \setminus subj / attr-rel]$,

to $\mapsto C_2 = [\#(\checkmark pre-TO-obj) / prep-wh]$,

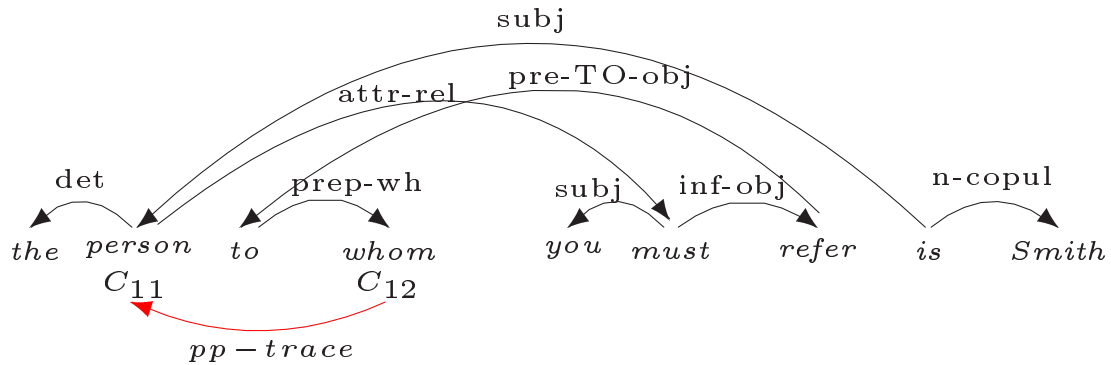
must $\mapsto C_3 = [subj \ \#(\checkmark pre-TO-obj) \setminus attr-rel / inf-obj]$,

refer $\mapsto C_4 = [(\checkmark pre-TO-obj) \setminus inf-obj]$,

is $\mapsto C_5 = [subj \setminus S / n-copul]$

III.1. Co-reference through cycles

EX 10 :

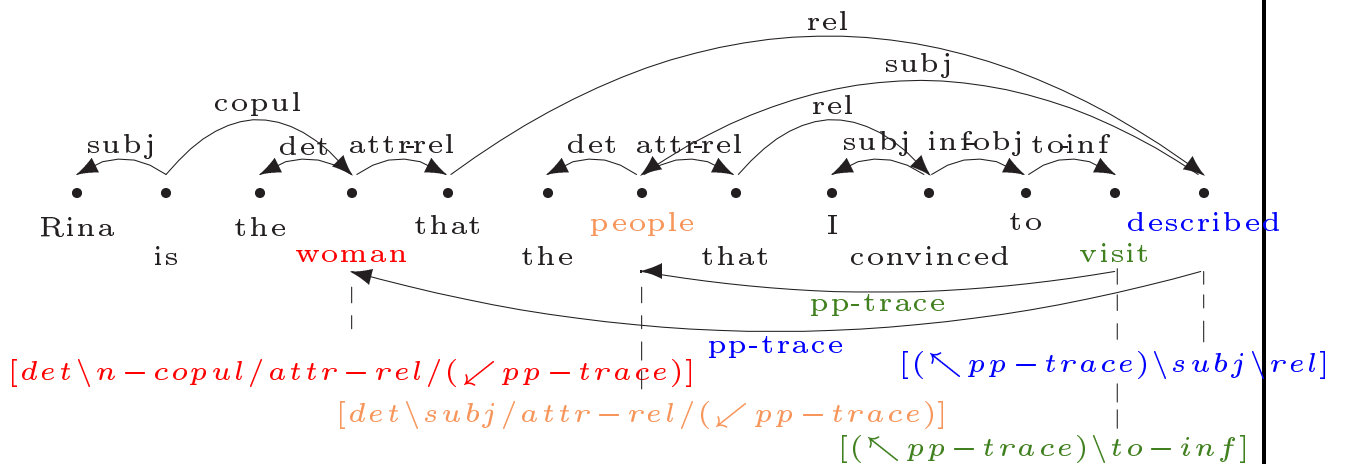


$$C_{11} = [det \setminus subj / attr-rel / (\surd pp-trace)],$$

$$C_{12} = [(\surd pp-trace) \setminus prep-wh]$$

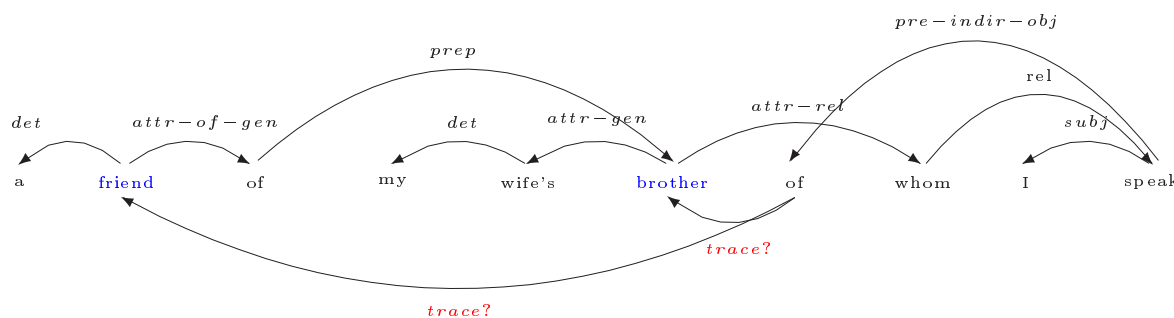
III.2. Pied-piping : still working

EX 11 (N. Vailette) :

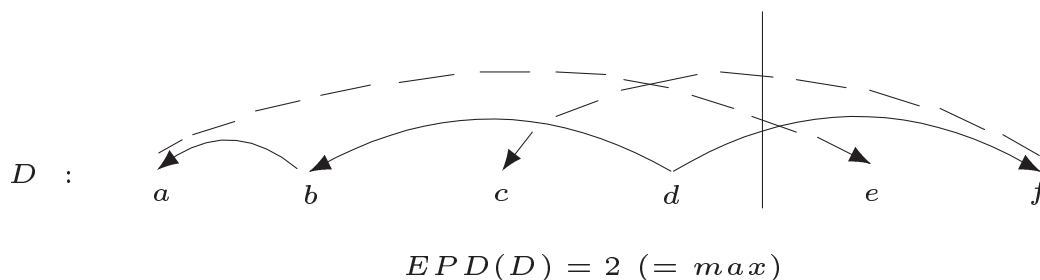


III.3. Where it is not working :

EX 12



III.4. Discontinuous dependencies' thickness (DDT)



TH If $DDT(G)$ is finite for $G \in GCD^{cs}$, then $L(G) \in \mathcal{L}(CF)$.

Note : In natural languages, DDT is bounded by a small constant (2-3).

III.5. CDG vs. CFG :

$G_0 = (\{a, b, c, d_1, d_2, d_3\}, \mathbf{C}_0, S, \delta_0) \in GCD^{FA}$:

$a \mapsto [\beta \setminus \alpha], [\alpha \setminus \alpha], \quad d_1 \mapsto \alpha,$

$b \mapsto [\alpha_1 \setminus D/A], \quad d_2 \mapsto [\alpha \setminus \beta_1 \setminus S/D],$

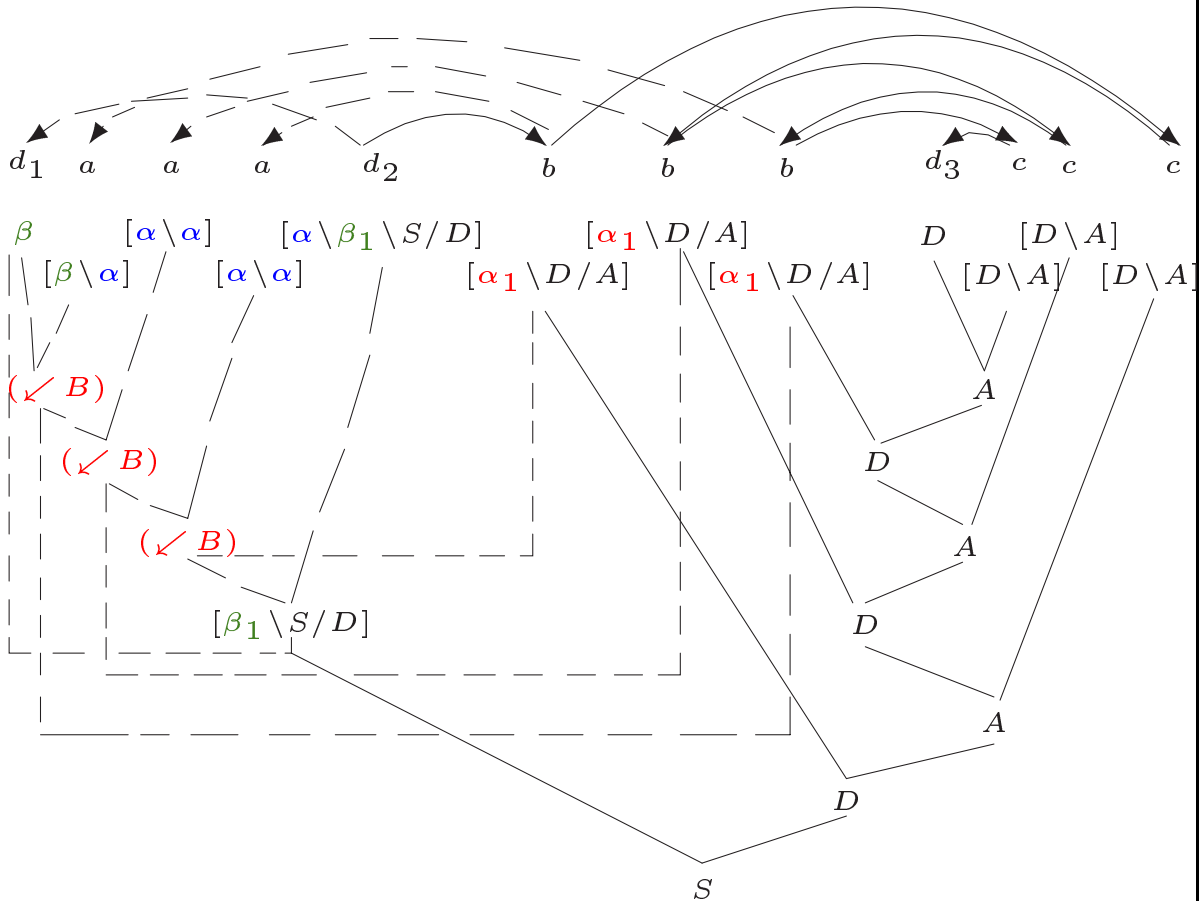
$c \mapsto [D \setminus A], \quad d_3 \mapsto D,$

$\alpha = \#(\swarrow B), \alpha_1 = (\nwarrow B), \beta = \#(\swarrow C),$

$\beta_1 = (\nwarrow C)$

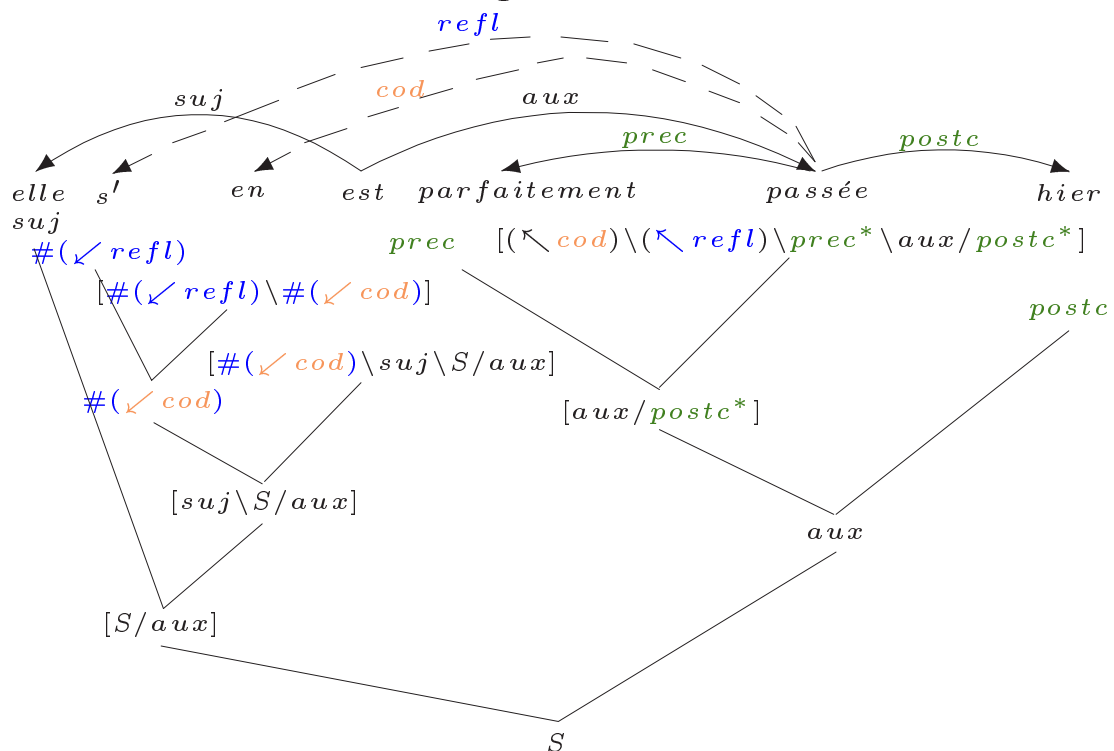
generates $L(G_0) = \{d_1 a^n d_2 b^n d_3 c^n \mid n > 0\}$.

A proof for $d_1 a^3 d_2 b^3 d_3 c^3$:



TH $\mathcal{L}(CF) \subsetneq \mathcal{L}(CDG)$.

EX 13 : Clitics through anchored types

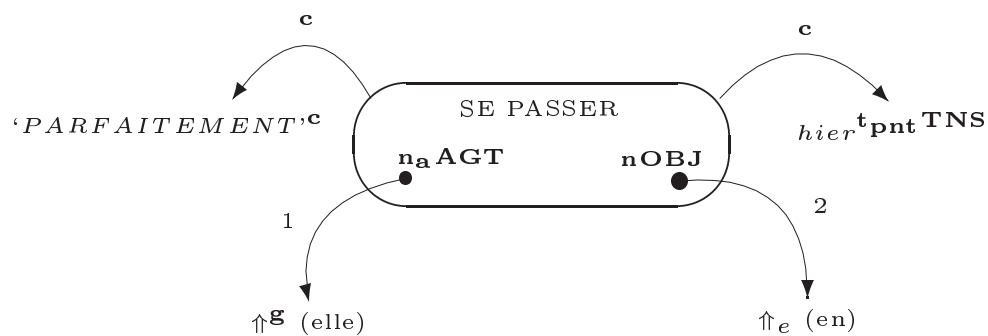


Types :

être \mapsto $[\#(\checkmark \text{ cod}) \setminus \text{subj} \setminus S/\text{aux}]$

en \mapsto $[\#(\checkmark \text{ refl}) \setminus \#(\checkmark \text{ cod})]$

se \mapsto $\#(\checkmark \text{ refl})$



III.6. CDG vs. TAG

TH For all $m > 4$, the language

$$\{d_0 a_0^n d_1 a_1^n \dots d_m a_m^n d_{m+1} \mid n \geq 0\} \notin \mathcal{L}(TAG)$$

is generated by the CDG $G_{(m)}$:

$$d_0 \mapsto [S/D_0] \text{ et } d_{m+1} \mapsto D_m,$$

$$a_0 \mapsto [D_0/D_0/(\nearrow A_m)/\dots/(\nearrow A_1)],$$

$$d_i \mapsto [D_{i-1}/\#(\searrow A_i)],$$

$$a_i \mapsto [\#(\searrow A_i)/\#(\searrow A_i)], [\#(\searrow A_i)/D_i]$$

$$(0 < i \leq m).$$

Hypothesis : The copy language

$$\{ww \mid w \in \{a, b\}^+\} \in \mathcal{L}(TAG)$$

is not generated by CDG.

III.7. CDG vs. multi-TAG

$$\mathfrak{3}_{mix} =_{df} \{w \in \{a, b, c\}^+ \mid |w|_a = |w|_b = |w|_c\}$$

Hypothesis : $\mathfrak{3}_{mix} \notin \mathcal{L}(multi - TAG)$.

TH (together with A. Foret) $\mathfrak{3}_{mix} \in \mathcal{L}(GCD)$.

IV. Complexity

TH Membership problem

$$\{(G, w) \mid G \in GCD^{sc}, w \in W^+\}$$

is NP-complete.

TH Parsing problem is polynomial ($\mathbf{O}(n^4)$ if DDT is finite, $\mathbf{O}(n^3)$ if G is projective).

Independent projections criterion :

Local projection : $\|\gamma\|_l$. Let $\gamma \in Cat(\mathbf{C})^*$.

11. $\|\varepsilon\|_l = \varepsilon$; $\|C\gamma\|_l = \|C\|_l \|\gamma\|_l$ for $C \in Cat(\mathbf{C})$ and $\gamma \in Cat(\mathbf{C})^*$.

12. $\|C\|_l = C$ for $C \in \mathbf{C} \cup \mathbf{C}^* \cup Anc(\mathbf{C})$.

13. $\|C\|_l = \varepsilon$ for $C \in V^+(\mathbf{C}) \cup V^-(\mathbf{C})$.

14. $\|[a \setminus \alpha]\|_l = [a \setminus \|\alpha\|_l]$ and $\|[\alpha / a]\|_l = [\|\alpha\|_l / a]$ for $a \in \mathbf{C} \cup \mathbf{C}^* \cup Anc(\mathbf{C})$ and $\alpha \in Cat(\mathbf{C})$.

15. $\|[(\curvearrowright a) \setminus \alpha]\|_l = \|[\alpha / (\curvearrowleft a)]\|_l = \|\alpha\|_l$ for all $a \in \mathbf{C}$ and $\alpha \in Cat(\mathbf{C})$.

Valencies' projection : $\|\gamma\|_v$

v1. $\|\varepsilon\|_v = \varepsilon$; $\|C\gamma\|_v = \|C\|_v\|\gamma\|_v$ for $C \in \text{Cat}(\mathbf{C})$ and $\gamma \in \text{Cat}(\mathbf{C})^*$.

v2. $\|C\|_v = \varepsilon$ for $C \in \mathbf{C} \cup \mathbf{C}^*$.

v3. $\|C\|_v = C$ for $C \in V^+(\mathbf{C}) \cup V^-(\mathbf{C})$.

v4. $\|\#(C)\|_v = C$ for $C \in V^-(\mathbf{C})$.

v5. $\|[\alpha]\|_v = \|\alpha\|_v$ for all $[\alpha] \in \text{Cat}(\mathbf{C})$.

v6. $\|\#(C)\backslash\alpha\|_v = \|\alpha/\#(C)\|_v = \|\alpha\|_v$.

v7. $\|\alpha_1\backslash\alpha_2\|_v = \|\alpha_1\|_v\|\alpha_2\|_v$, if $\alpha_1 \notin \text{Anc}(\mathbf{C})$.

v8. $\|\alpha_1/\alpha_2\|_v = \|\alpha_1\|_v\|\alpha_2\|_v$, if $\alpha_2 \notin \text{Anc}(\mathbf{C})$.

TH Let $G = (W, \mathbf{C}, S, \delta)$ be a CDG.

$x \in L(G)$ iff there is a string of categories $\alpha \in \delta(x)$ and its **realization** $\gamma \in R(\alpha)$ such that :

1. $\|\gamma\|_v \vdash_p^* S$,
2. there is a correct pairing of dual categories $(\alpha, \check{\alpha})$ in $\|\gamma\|_v$.

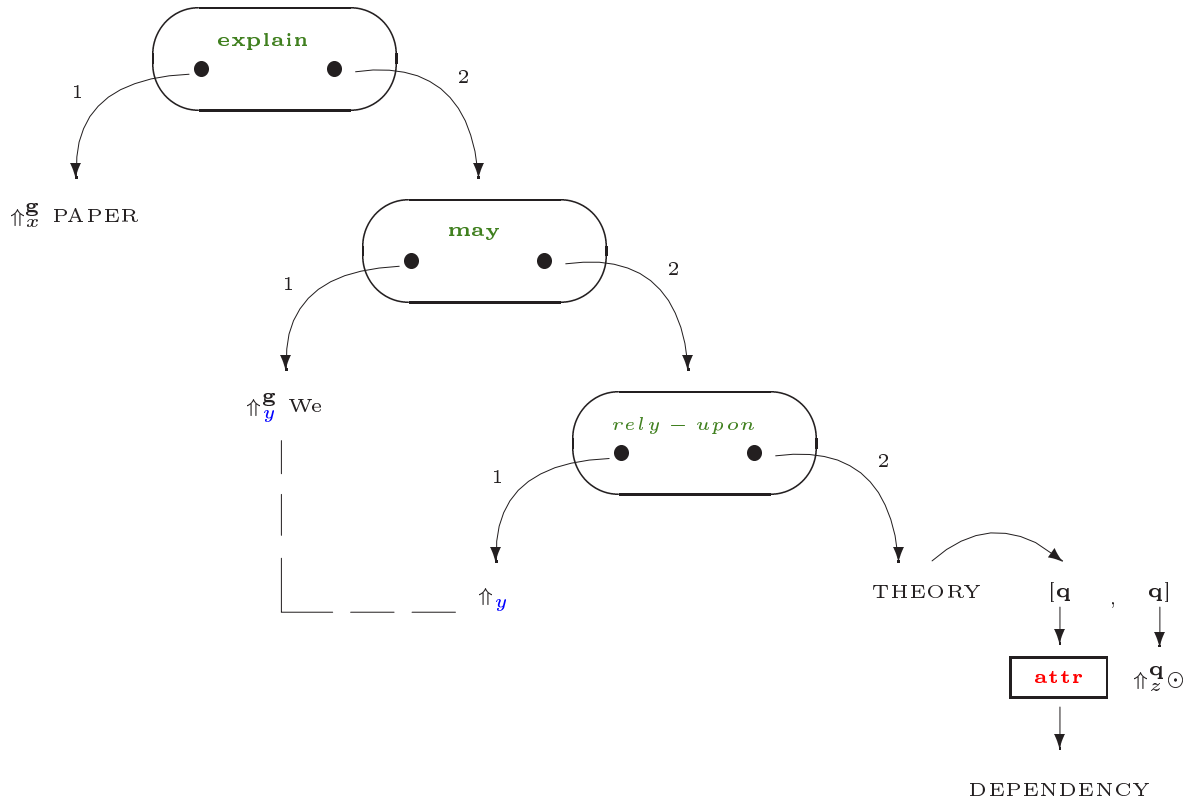
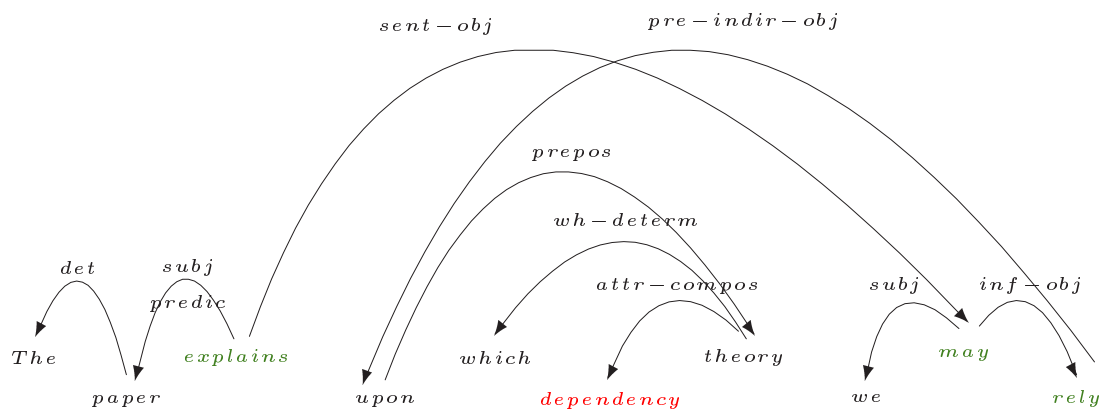
For $C = [\alpha D^* \backslash \beta]$, $[\alpha \beta]$, $[\alpha D \backslash \beta]$, $[\alpha D \backslash D \backslash \beta]$, etc. are its **realizations**.

\vdash_p^* : elimination-rule (**L**) provability.

V. Problematic :

1. Translation of constraint dependency languages into CDG.
2. Sound type translation expansion to complete resource-logic semantics via situational semantics.
3. Robust (linear-time) incremental parser with a stochastic oracle.
4. Modular generation of CDG types from situational meanings.

EX 10 :



III.2. Comment éviter les cycles

1. Pour le calcul non-commutatif $\mathbf{CD}^{\mathbf{FA}}$:

TH $G \in CDG^{FA} \wedge D \in D(G) \Rightarrow (D \text{ est } AD)$.

2. Pour $\mathbf{CD}^{\mathbf{sc}}$: contraintes sur les types.

GCD bien fondée :

$\gamma_1 \rightsquigarrow_l \gamma_2$: γ_2 “est dominée par” γ_1 (à travers les dépendances locales ou distantes) ;

$\gamma_1 \hookrightarrow \gamma_2$: $[\alpha \setminus \gamma_1 / \beta_1 \gamma' \beta_2] \in \delta(W)$ et $\gamma' \rightsquigarrow \gamma_2$.

BF : Il y a un préordre \succsim sur Cat :

w₁. If $\alpha_1 \rightsquigarrow \alpha_2$, then $\alpha_1 \succsim \alpha_2$.

w₂. If $(\swarrow A) \hookrightarrow (\nearrow B) \rightsquigarrow \alpha$ and $(\nwarrow A) \leftarrow \alpha$, then $(\swarrow A) > (\nearrow B)$.

TH Si $G \in CDG^{sc}$ est WF et $D \in D(G)$, alors D est AD.